MATH 114 ASSIGNMENT 13, FALL 2015

Due in class on Friday, December 4.

Part 1. Do and write up the following problems.

1. [240s10] Math 240 Final exam, May 4, 2010, problem 13.
2. [240s10] Math 240 Final exam, May 4, 2010, problem 14.
3. [240f08] Math 240 Final Exam, December 17, 2008, problem 6.
4. [240f08] Math 240 Final Exam, December 17, 2008, problem 7.
5. [240f09] Math 240 Final Exam, Dec 21, 2009, problem 9.
6. [240f09] Math 240 Final Exam, Dec 21, 2009, problem 10.
7. [240f09] Math 240 Final Exam, Dec 21, 2009, problem 11.
8. [240f04] Math 240 Final Exam, Fall 2004, problem 8.
9. [240f04] Math 240 Final Exam, Fall 2004, problem 9.

Part 2. A few more problems for you to practice. (They are not part of this assignment.)
A. Let C be the boundary of the square

$$Q := \{(x,y) \mid -1 \le x, y \le 1\}$$

on the plane, oriented counterclockwise. Compute the line integral

$$\oint_C (y^2 + \cos(x^2))\, dx + (x + \sin(y^2))\, dy$$

B. Compute the oriented surface integral

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{N}\, d\sigma$$

where

$$\vec{F}(x,y,z) = (y+x)\vec{i} + (y-x)\vec{k} + \sin(xyz)\vec{k},$$

and S is the surface

$$S = \{(x,y,z) \mid z = x^2 + y^2 - 9,\ z \le 0\}$$

oriented by the continuous unit normal vector field $\vec{N}$ on S such that $\vec{n}(0,0,-9) = -\vec{k}$.

C. Let $\vec{F}(x,y) = xy\vec{i} + xe^{-y\cos z}\vec{j} + \frac{e^x - e^{-x}}{2x} yz\vec{k}$. Compute the triple integral

$$\iiint_V \operatorname{div}(\operatorname{curl}(\vec{F}))\, dx\, dy\, dz$$

where V is the unit sphere $\{x^2 + y^2 + z^2 \le 1\}$.

D. For what values of the parameter λ is

$$\vec{F}(x,y) = -6x \sin y\, \vec{i} + (\lambda^2 - 4)x^2 \cos y\, \vec{j}$$

a conservative vector field? For such values of λ, compute the line integral

$$\oint_C \vec{F} \cdot d\vec{r},$$

where C is the straight line segment from $(1,0)$ to $(0,1)$.

E. Let C be the rectangle whose vertices are $(-2,-3)$, $(2,-3)$, $(2,3)$ and $(-2,3)$, oriented counter-clockwise. Compute the line integral

$$\oint_C x^2 y\,dx + y^3 x^3\,dy\,.$$

F. Let $\vec{F} = \vec{F}(x,y,z)$ be a smooth vector field defined on $V = \{1 \leq x^2+y^2+z^2 \leq 100\}$. Let $\vec{N}(x,y,z) = \frac{x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$. Which ones among the following statements are true?

(I) If $\vec{F} = \mathrm{curl}(G)$ for a smooth vector field G defined on V, then $\mathrm{div}(\vec{F}) = 0$.

(II) If $\mathrm{div}(\vec{F}) = 0$, then there exists a smooth vector field G such that $\vec{F} = \mathrm{curl}(G)$.

(III) If $\mathrm{div}(\vec{F}) = 0$, then

$$\iint_{\{x^2+y^2+z^2=100\}} \vec{F}\cdot\vec{N}\,d\sigma = 0\,.$$

(IV) If $\mathrm{div}(\vec{F}) = 0$, then

$$\iint_{\{x^2+y^2+z^2=100\}} \vec{F}\cdot\vec{N}\,d\sigma = \iint_{\{x^2+y^2+z^2=1\}} \vec{F}\cdot\vec{N}\,d\sigma\,.$$

(V) If $\vec{F} = \mathrm{curl}(G)$ for a smooth vector field G, then

$$\iint_{\{x^2+y^2+z^2=100\}} \vec{F}\cdot\vec{N}\,d\sigma = 0\,.$$