MATH 114 ASSIGNMENT 2, FALL 2015

Due in class on Friday, September 11th

A large part of this assignment are from Math 114 final exams in recent years. You will get a good idea about the level of proficiency expected for this course when you work through these old exam problems.

Part 1. Read Thomas 13.1 to 13.6 and make sure you can do all core problems in these three sections.

Part 2. Do and write up the following problems, from Thomas and old final exams.

12.6 Exercise 46.

13.1 Exercise 24.

13.2 Exercise 31 (a). (Hint: You know the general form of the trajectory of a projectile under a uniform/constant gravitational force $\mathbf{g}$, which is a function of the time t; there are some parameters in this formula which depends on the particular trajectory in question. In the present situation, you can express these parameters in terms of the angle α and the inclination of the hill. Here the inclination is the angle BOR, where the half-line from O to R is in the horizontal direction and lies inside the angle AOR.)

13.3 Exercise 17 (a) and (d).

F13 Math 114 Final Exam, December 17, 2013, Problems 1 and 2.

S14 Math 114 Final Exam, May 5, 2014, Problem 1.

S13 Math 114 Final Exam, Spring 2013, Problems 1, 2.

F12 Math 114 Final Exam, Fall 2012, Problems 1, 3.

Part 3. Extra credit problems.
E1. (a) Show that the cross product satisfy the following property:

$$A(\mathbf{u}\times\mathbf{v}) = A\mathbf{u}\times A\mathbf{v} \quad \text{for all } \mathbf{u},\mathbf{v}\in\mathbb{R}^3$$

for every 3×3 matrix A with coefficients in $\mathbb{R}$ such that $A\cdot A^T = \mathrm{I}_3 = A^T\cdot A$ and $\det(A)=1$.
(This is the property that cross product is invariant under rotations we discussed in class. Square matrices A with real entries such that whose inverse is its transpose are called *orthogonal matrices*. A rotation is an orthogonal matrix with determinant 1.)

(b) Is the cross product invariant under all 3×3 orthogonal matrices?

E2. Consider three "variable vectors" $\mathbf{u} = u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k}$, $\mathbf{v} = v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k}$ and $\mathbf{w} = w_1\mathbf{i}+w_2\mathbf{j}+w_3\mathbf{k}$, where the coefficients $u_1,u_2,u_3,v_1,v_2,v_3,w_1,w_2,w_3$ are variable.

(a) Show that there are homogenous cubic polynomials

$$a(v_1,v_2,v_3,w_1,w_2,w_3) =: a(\mathbf{v},\mathbf{w}) \quad \text{and} \quad b(u_1,u_2,u_3,v_1,v_2,v_3) =: b(\mathbf{u},\mathbf{w})$$

such that

$$(\mathbf{u}\times\mathbf{v})\times\mathbf{w} = a(\mathbf{v},\mathbf{w})\mathbf{u} + b(\mathbf{u},\mathbf{w})\mathbf{v}$$

(b) Show that $a(\mathbf{v},\mathbf{w}) = -\mathbf{v}\cdot\mathbf{w}$ and $b(\mathbf{u},\mathbf{w}) = \mathbf{u}\cdot\mathbf{w}$.

[Hint: The rotational symmetry of the cross product may help.]