MATH 114 ASSIGNMENT 3, FALL 2015

Due in class on Friday, September 18th

We have assigned just a few exercises from the book. However you will likely find just doing them does not provide enough opportunity to know the materials well enough. It is a good idea to do more exercises on your own. And if you have access to MyMathLab, you may want to try the extra credit problems there (easier than the extra credit problem below) and also some more problems on your own.

Part 1. Read Thomas 13.4 to 14.2.

Part 2. Do and write up the following problems, from Thomas and old final exams.

13.4 Exercise 19.

13.5 Exercise 14.

13.6 Exercise 4.

S13 Math 114 Final Exam, Spring 2013, Problems 3, 4, 5.

F12 Math 114 Final Exam, Fall 2012, Problems 4, 5.

S11 Math 114 Final Exam, Spring 2011, Problem 2.

Part 3. Extra credit problems.

This is a continuation of the extra credit problem in HW 1. There you learned about the addition and multiplication of Hamiltonian quaternions.

(Recall that a typical element of Hamiltonian quaternion is of the form $u+x\cdot\vec{i}+y\cdot\vec{j}+z\cdot\vec{k}$ with $u,x,y,z\in\mathbb{R}$ You may think such a quaternion is the sum of its "real part" u and it "imaginary part" $x\cdot\vec{i}+y\cdot\vec{j}+z\cdot\vec{k}$. The "imaginary part" (respectively "real part") of the product

$$(x_1\cdot\vec{i}+y_1\cdot\vec{j}+z_1\cdot\vec{k})\cdot(x_2\cdot\vec{i}+y_2\cdot\vec{j}+z_2\cdot\vec{k})$$

of two "purely imaginary" quaternions is the cross product (respectively the dot product) of the two corresponding vectors in $\mathbb{R}^3$.)

E1. Show that for every non-zero quaternion q has an inverse, i.e. a quaternion p such that $p\cdot q = q\cdot p = 1$. Show moreover that the above property uniquely determines *the* inverse of p, i.e. if p_1 and p_2 are both inverses of q, then $p_1 = p_2$. The inverse of q is usually written as q^{-1}.

E1 Let q be a non-zero quaternion. Show that the set of all purely imaginary quaternions is stable under the map/function which sends every quaternion p to $q\cdot p\cdot q^{-1}$; in other words $q\cdot p\cdot q^{-1}$ is purely imaginary if p is.

E3 The map $p\mapsto q\cdot p\cdot q^{-1}$ in E2 for purely imaginary quaternions p can be thought of as a map from $\mathbb{R}^3$ to $\mathbb{R}^3$. Identify it geometrically.

[Hint: Is this map a translation? A rigid motion in $\mathbb{R}^3$ such as a rotation? Rotating a certain angle with respect to a certain axis?]