MATH 114 ASSIGNMENT 4, FALL 2015

Due in class on Monday, September 28th

Part 1. Read Thomas 14.2 to 14.4.

Part 2. Do and write up the following problems, from Thomas and old final exams.

14.2 Exercise 34

14.3 Exercise 48

14.4 Exercise 40, 42, 48

S11 Math 114 Final Exam, Spring 2011, problem 4.

S12 Math 114 Final Exam, May 4, 2012, problems 4, 6.

F13 Math 114 Final Exam, December 17, 2013, problem 3.

S14 Math 114 Final Exam, May 5, 2014, problem 5.

Part 3. Extra credit problem.

Suppose that we have a surface S in $\mathbb{R}^3$, which can be described by any one of the following three *equivalent* ways

$$z = f(x,y), \quad x = g(y,z), \quad \text{or} \quad y = h(x,z)$$

for three smooth functions $f(x,y), g(y,z)$ and $h(x,z)$. Suppose moreover we have a function F on S. This function F can be thought of as a function in the variables x,y, i.e $F(x,y,z(x,y))$. On the other hands, writing x as a function in y and z, we can think of F as $F(g(y,z),y,z)$, a function in the variables y and z. Similarly we can think of F as a function in x,z.

In the present situation there are two partial derivatives of F with respect to y:

$$\frac{\partial}{\partial y}F(x,y,z(x,y)) \quad \text{and} \quad \frac{\partial}{\partial y}F(g(y,z),y,z),$$

often abbreviated as $\left(\frac{\partial F}{\partial y}\right)_x$ and $\left(\frac{\partial F}{\partial y}\right)_z$ respectively. These two partial derivatives are quite different. Find a relation between these two partial derivatives. (The relation you find should involve at least one of f, g and h. Of course the functions f, g and h are closely related.)

[In the above the variables x,y,z are related to each other so that there are only two "free variables"; this is a familiar situation in thermodynamics. Any one of f,g,h is said to be an *equation of state*.]