MATH 114 ASSIGNMENT 9, FALL 2015

Due in class on Friday, November 6.

Orientation of curve and surfaces is an important part of oriented surface integrals. This is also a part which get in the way when people try to understand Stokes' theorem. We explained orientation in class in the week before the second midterm. You can also consult §2 of notes posted on the math 114 page. The problems below are exercises in §2 of the notes, so that you can practice with the concept of orientation in a setting separate from Stokes' and the divergence theorems.

Part 1. Read §2 of the notes, on orientation of curves and surfaces. Read also the explanation of orientation of a surface on page 1016 of Thomas and compare with the definition given in class and in the notes.

Part 2.

1. Let D be the solid

$$D := \{(x,y,z) \in \mathbb{R}^3 \,|\, x,y,z \geq 0,\ 0 \leq x+y+z \leq 3\}$$

   The boundary ∂D of D is a closed surface, and is a union of 4 triangles. Let $\vec{N}$ be the unit normal vector field on ∂D pointing *away* from D.

   (a) Give an explicit formula for the normal vector field $\vec{N}$.

   (b) Let $S := \{(x,y,z) \in \partial D \,|\, x+y+z \geq 1\}$. The restriction of $\vec{N}$ to S is an orientation of S, which induces an orientation on the boundary ∂S of S. Describe explicitly the closed curve ∂S and the orientation of ∂S induced by the orientation $\vec{N}$ of S.
   [Note that ∂S is a triangle.]

2. Let C be the boundary of the surface

$$S = \{(x,0,z) \,|\, 1 \leq x^2 + 4z^2 \leq 4\}$$

   on the (x,z)-plane. Note that C is a disjoint union of two ellipses on the (x,z)-plane. Orient S by the normal vector field $\vec{j}$. The orientation $\vec{N}$ of S defines an unit tangenet vector field $\vec{T}$, which gives C an orientation. Find $\vec{T}(1,0,0)$ and $\vec{T}(0,0,1)$.

3. Let S be the surface

$$S := \left\{ (x,y,z) \in \mathbb{R}^3 \,\middle|\, x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 36,\ x-y-z+10 \geq 0 \right\}$$

   Let $\vec{N}$ be the continuous unit normal vector field on S such that $\vec{N}(6,0,0) = \vec{i}$. Let $C := \partial S$ be the boundary of S, and let $\vec{T}$ be the unit tangent vector field on C giving C the orientation induced by $(S,\vec{N})$. Compute $\vec{T}(4,8,6)$.

   (Note that $(6,0,0)$ is a point of the surface S and $(4,8,6)$ is a point of the curve C.)

4. Let S be the surface

$$S := \left\{ (y^2+yz+z^2, y, z) \in \mathbb{R}^3 \,\middle|\, -10 \leq y,z \leq 10,\ (y-2)^2 + z^2 \geq 1,\ y^2 + (z-2)^2 \geq 1 \right\}.$$

You can think of S as the graph of the function

$$(y,z) \quad \mapsto \quad y^2 + yz + z^2$$

on the region of the (y,z)-plane consisting of all point lying inside a square edge length 20 and outside two circles of radius 1. Let $\vec{N}$ be the continuous unit vector field on S such that $\vec{N}(0,0,0) = -\vec{i}$. The boundary ∂S is the union of three piecewise smooth closed curves. Let $\vec{T}$ be the unit tangent vector field on the smooth locus of ∂S giving the orientation of ∂ induced by the oriented surface $(S, \vec{N})$. Compute $\vec{T}(100,10,0)$, $\vec{T}(9,3,0)$ and $\vec{T}(9,0,3)$.

(The three points $(100,10,0), (9,3,0)$ and $(9,0,3)$ lie on the three connected components of ∂S respectively.)

Part 3. Extra credit problems.

E1. Let D be the solid in $\mathbb{R}^3$ obtained by rotation the disk

$$\{(x,z) \,|\, (x-5)^2 + z^2 \leq 4\}$$

in the (x,z)-plane about the z-axis. Let ∂D be the boundary of D, which is a torus, oriented by the unit normal vector field $\vec{N}$ on ∂D pointing *away* from D.

(a) Let ∂D_- be the lower-half of ∂D, consisting of those points of ∂D whose z-coordinates are non-positive. Describe the projection of ∂D_- to the (x,y)-plane *explicitly*. (This is a region R in the (x,y)-plane.)

(b) The surface ∂D_- is the graph of a function $f : R \to \mathbb{R}$. Find this function f and write down the corresponding parametrization $\vec{r} = x\vec{i} + y\vec{j} + f\vec{k}$ of ∂D_-.

(c) Determine the sign function $(\partial D_-, \vec{N}, \vec{r})$ of for the parametrization $\vec{r}$ in (b) of the oriented surface $(\partial D_-, \vec{N})$.

E2. Let D be the solid in $\mathbb{R}^3$ obtained by rotation the disk

$$\{(x,z) \,|\, (x-5)^2 + z^2 \leq 4\}$$

in the (x,z)-plane about the z-axis. Let ∂D be the boundary of D, which is a torus, oriented by the unit normal vector field $\vec{N}$ on ∂D pointing *away* from D. Let S be the surface

$$\{(x,y,z) \in \partial D \,|\, 0 \leq x \leq 6\}.$$

Let $\vec{T}$ be the unit tangent vector field on the boundary ∂S of S induced by the oriented surface $(S, \vec{N})$. Note that ∂S is a disjoint union of two circles C_1, C_2 of radius 2 on the (y,z)-plane and a closed curve C_3 on the plane $\{x = 6\}$.

(a) Describe the two cicles C_1, C_2 and the unit tangent vector field $\vec{T}$ on them explicitly.

(b) Find $\vec{T}(6,0,\sqrt{3})$.