Math 240

Short Answers/Solutions to Practice Problems. February 2015

1. (a) The general solution is $\begin{pmatrix} 3 \\ -4 \\ 2 \\ 0 \end{pmatrix} + x_4 \cdot \begin{pmatrix} -1 \\ 3 \\ -1 \\ 1 \end{pmatrix}$

The two solutions with $x_3 \cdot x_4 = 0$ are $\begin{pmatrix} 3 \\ -4 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}$

(b) $\det \begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 3 & 2 \\ 1 & 0 & 0 & 1 \\ 2 & 1 & 2 & 1 \end{pmatrix} = 0$

2. Ans. Except the point $(2,1,1)$, the other six points all lie on the plane $\{3x - 7y - 2 = 0\}$

3. (a) $\det$ (This 8×8 matrix) $= 1$

(b) $\det$ (The 6×6 matrix) $= 3$

4. Ans. $B^{-1} = \begin{pmatrix} -2 & 5 & -3 \\ -8 & 17 & -10 \\ 5 & -10 & 6 \end{pmatrix}$

5. (a) The rank is 3 (almost already in reduced echelon form)

(b) The general solution is: $x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_6 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}$

6. (a) The linear span of the columns are $\left\{ \begin{pmatrix} u \\ v \\ 2u - 3v \end{pmatrix} : u, v \in F \right\}$

$\dim$ (linear span of columns) $= 2$

(b) $c = -11$

7. (a) $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $AB \neq BA$

(b) $C = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $C^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(c) $B_1 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$, $B_2 = (\vec{e}_1, \vec{e}_3, \vec{e}_2)$, $B_3 = (\vec{e}_1 + \vec{e}_2, \vec{e}_1 - \vec{e}_2, \vec{e}_3)$

7. (d) $T : \left\{ \begin{matrix} \text{all } \mathbb{R}\text{-valued} \\ \text{functions on } \mathbb{R} \end{matrix} \right\} \longrightarrow \left\{ \begin{matrix} \text{all } \mathbb{R}\text{-valued} \\ \text{functions on } \mathbb{R} \end{matrix} \right\}$

$f(x) \longmapsto x^2+1$ for every function $f(x)$ on $\mathbb{R}$

(e) $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

8. (a) a basis of Q : $(1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3)$

(b) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 \end{bmatrix}$

(c) $\det \begin{pmatrix} \text{this } 10\times 10 \\ \text{matrix} \end{pmatrix} = 0$

(d) $\mathrm{Ker}(T) = F$ (= all constant polynomials)

(e) $\dim_F(\mathrm{Im}(T)) = 9$

9. (a) F (S is not a vector subspace $\because$ $0 \notin S$)

(b) T ($\because$ $\dim_F \mathrm{Im}(T) \leq 10 < 12 = \dim_F(P_{12})$)

(c) F e.g. $\det \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} = 1$

(d) F

(e) T

(f) F ($\det(2\cdot A) = 2^5 \det(A)$ since A is a 5×5 matrix)

(g) T

(h) F ($B \cdot A$ is invertible $\Rightarrow$ $rk(A) = 6$ $\Rightarrow$ the 6 columns of A are linearly independent.