MATH 240 ASSIGNMENT 10, SPRING 2015

Due in class on Monday, April 20, 2015

: Note: This is the last homework assignment; the due date is the Monday after the "usual" Friday.
Part A. (a) Read DELA §7.9, §7.10

Part B. Do the following problems in DELA.

§7.9 Problems 14, 22, 24, 25, 26

§7.10 Problems 6, 8, 11, 12, 20

B1. Determine all equilibrium points of the non-linear autonomous system

$$\begin{aligned} \frac{dx}{dt} &= y(x-4) \\ \frac{dy}{dt} &= x-4y^2 \end{aligned} \tag{1}$$

and describe the type of each equilibrium point as
A) stable node B) unstable node C) saddle point D) center E) stable spiral F) unstable spiral

B2. Find all equilibrium points of the following non-linear autonomous system

$$\begin{aligned} \frac{dx}{dt} &= x+y-x(x^2+y^2) \\ \frac{dy}{dt} &= -x+y-y(x^2+y^2) \end{aligned} \tag{2}$$

and determine their type.

B3. (extra credit) Investigate the non-linear autonomous system in B2.

(a) Draw a phase portrait. It is a good idea to use a computer algebra system (such as Maple, Mathematica or Matlab) if one is available. The phase portrait would make the statement (c) below look plausible.

(b) Show that in the polar coordinate system (r,θ) for $\mathbb{R}^2$, the system of equations (2) is equivalent to

$$\begin{aligned} r\frac{dr}{dt} &= r^2(1-r^2) \\ \frac{d\theta}{dt} &= -1 \end{aligned} \tag{3}$$

(c) Show/explain that there is no *closed* trajectory (integral curve) inside the unit disk centered about the origin.

(b) Show/explain that every trajectory outside the unit circle spirals toward the unit circle.