MATH 240 ASSIGNMENT 2, SPRING 2015

Due in class on Friday, January 30

Part 0. Read §3.4 of DELA. Note: Although the cofactor expansion is not in the syllabus, it is something good to know. The formula of the inverse matrix (called the *adjoint method* in the book) and Cramer's rule/formula for the solution of a system of linear equations when the matrix in question is invertible are both (essentially) equivalent to the cofactor expansion.

Part 1. Do (but do not hand in) the following problems from DELA:

§2.6 Problems 26, 31, 34

§3.2 Problems 53, 54

§3.4 Problems 19

Part 2. Do and write up the following problems from DELA:

§2.6 Problems 16, 37

§3.1 Problems 24, 36

§3.2 Problems 8, 10, 22

§3.4 Problem 6

Part 3. Extra credit problems:

(i) §3.2, Problem 58 of DELA

(ii) Say you have k linear algebraic equations in n variables; in matrix form we write $AX = Y$. Give a proof or a counterexample for each of the following statements.

- (a) If $n = k$ there is always at most one solution.
- (b) If $n > k$ the equation $AX = Y$ has a solution for any given Y.
- (c) If $n > k$ the nullspace of A has dimension greater than zero.

(iii) The $n \times n$ Hilbert matrix H_n is defined in problem 44 of §2.6; its (i, j) entry is $\frac{1}{i+j-1}$. It is a fact that $\det(A_n)$ is a positive rational number of the form $\frac{1}{C_n}$ for some positive integer C_n.

- (a) Verify the above statement for $n = 4,5,6$.
- (b) Try to prove this statement.
  [If you succeed in proving this statement, or even the weaker statement that $\det(H_n) \neq 0$ for every n, please come to my office and show me your proof.]