MATH 240 ASSIGNMENT 5, SPRING 2015

Due in class on Friday, February 27

Part 1.

(a) Read DELA §5.6, §5.7, §5.8, §5.9, §5.11

Note 1. Although §5.7 is not included in the syllabus, its content is both useful and helpful for understanding eigenvalues and eigenvectors. Jordan forms (§5.11), optional in the syllabus, will not be included for the final exam. (The terminology "non-defective" used in the book is not standard.)

Note 2. A square matrix A is *non-defective* as defined in §5.7 if and only if A is *diagonalizable* as defined in §5.8. In other words these two notions are equivalent.

Note 3. The definition of *characteristic polynomial* of an $n \times n$ matrix A used in the book is *different* from the standard usage. The standard definition of the characteristic polynomial of A is $\det(\lambda \cdot \mathrm{I}_n - A)$, while the book defines the characteristic polynomial as

$$\det(A - \lambda \cdot \mathrm{I}_n) = (-1)^n \det(\lambda \cdot \mathrm{I}_n - A).$$

(b) Do (but do not hand in) the following problems from DELA:

§5.6 T/F Review 2, 7, 8, 9 ; Problems 36, 37, 38

§5.7 T/F Review 1, 2, 5, 6; Problems 3, 4, 5

§5.8 Problems 6, 25, 26, 30

§5.9 T/F Review 2, 3, 5, 6; Problems 4, 5, 6, 23

Part 2. Do and write up the following problems.

2A From DELA:

§5.6 Problems 26, 28, 30

§5.8 Problems 8, 14, 24

§5.9 Problems 12, 16, 18

2B Let P_4 be the vector space over $\mathbb{R}$ consisting of all polynomials in a variable x of degree at most 4. Let $T : P_4 \to P_4$ be the linear transformation which sends every element $f(x) \in P_4$ to $x^2 \frac{d^2 f}{dx^2} + \frac{df}{dx}$.

(1) Is T diagonalizable? Find a basis of P_4 consisting of eigenvectors if T is diagonalizable. Otherwise show why T is not diagonalizable.

(2) Compute e^{tT}, where t is a variable (unrelated to T). You may want to use a basis of P_4 and represent T by a square matrix.

Part 3. Extra credit problems. You will receive partial credit after doing at least two examples with n at least 4. Please show me your solution if you succeed in solving either one of the two problems for infinitely many n's.

E1. Let n be a positive integer. Let $C_n = (c_{ij})_{1 \le i,j \le n}$ be the $n \times n$ matrix such that $c_{ij} = 1$ if $|i-j| = 1$, or if $(i,j) = (1,n)$, or if $(i,j) = (n,1)$. All other entries of C_n are 0. For instance

$$C_5 = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Note that the non-zero entries of C_n are those which are immediately above or below the diagonal, plus the upper right and the lower left corner. Find all eigenvalues and the corresponding eigenvectors of C_n, for all n.

E2. Let n be a positive integer. Let $U_n = (u_{ij})_{1 \le i,j \le n}$ be the $n \times n$ matrix such that $u_{ij} = 1$ if $|i-j| = 1$, or if $(i,j) = (1,n)$, or if $(i,j) = (n,1)$. All other entries of U_n are 0. For instance

$$U_5 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Find all eigenvalues and the corresponding eigenvectors of U_n, for all n.