MATH 240 ASSIGNMENT 7, SPRING 2015

Due in class on Friday, March 20

Note: (1) The second midterm will cover materials up to and including §6.7 (variation of parameters). Jordan forms (§5.11) is not included, in the sense that you will not be ask to find the Jordan form of a matrix. We did explain it briefly in class, and you can certainly use it in class. (For instance Jordan form is handy when computing matrix exponentials.) The materials in §6.9 is not in the second midterm.

(2) Hints for preparing the second midterm exam:

1. Do the practice problems.
2. Do old exam problems covered in this exam.
3. Try the challenge problems #38–47 on the Math 240 course page.

Part 1.

(a) Read DELA §§6.4, 6.5 6.7, 6.9

(b) Do (but do not hand in) the following problems from DELA:

§6.5 T/F Review 4, 5, 6; Problems 16, 25, 29, 33

§6.7 T/F Review 2 ; Problems 9, 11, 13, 31, 33,

§6.9 T/F Review ; Problems 3, 5, 8

Part 2. Do and write up the following problems from DELA:

§6.4 Problems 6, 8, 10

§6.5 Problems 24, 28, 30, 32

§6.7 Problems 8, 18, 20

§6.9 Problems 12, 14

Part 3. Extra credit problem.

E. Suppose that A is an $n \times n$ real matrix such that $A^2 = \mathrm{I}_n$.

(a) Show that if $\vec{x} \in \mathbb{R}^n$ is a vector in $\mathbb{R}^n$ such that $(A + \mathrm{I}_n) \cdot \vec{x} = \vec{0}$ and $(A - \mathrm{I}_n) \cdot \vec{x} = \vec{0}$. Then $\vec{x} = \vec{0}$.

(b) Show that $\dim(\mathrm{Ker}(A + \mathrm{I}_n)) + \dim(\mathrm{Ker}(A - \mathrm{I}_n)) = n$.

(c) Conclude from (b) that there exists an invertible $n \times n$ matrix C such that $C^{-1} \cdot A \cdot C$ is a diagonal matrix.