MATH 350 ASSIGNMENT 1, FALL 2015

Due in class on Friday, January 22nd

1. Show that
$$\bigcap_{n\in\mathbb{N}_{>0}} (-\tfrac{1}{n}, \tfrac{1}{n}) = \{0\}.$$
(Recall that $(-\frac{1}{n}, \frac{1}{n}) := \{x \in \mathbb{R} \mid -\frac{1}{n} < x < \frac{1}{n}.$)

2. Let X, Y, Z be sets and let $f : X \to Y$ and $g : Y \to Z$ be functions.
   (a) Suppose that f and g are both *onto* (or equivalently, *surjective*). Show that $g \circ f$ is onto.
   (b) Suppose that f and g are both *one-to-one* (or equivalently, *injective*). Show that $g \circ f$ is injective.
   (c) Suppose that $g \circ f$ is onto. Show that g is onto.
   (d) Suppose that $g \circ f$ is one-to-one. Show that f is one-to-one.
   (e) Suppose that $g \circ f$ is bijective (i.e. it is both one-to-one and onto), f is surjective, and g is injective. Is it true that f is bijective? (Either give a proof or a counter-example.)

3. Define a relation R on $\mathbb{Z}$ by
$$(a,b) \in R \iff \exists n \in \mathbb{Z} \text{ such that } a - b = 101 \cdot n$$
   (a) Prove that R is an equivalence relation.
   (b) Describe the set of all equivalence classes for R. In particular determine the number of equivalence classes for R.

4. It is a fact that there exists rational numbers a, b, c, d, e such that
$$\sum_{1\le i\le n} i^3 = an^4 + bn^3 + cn^2 + dn + e$$
for all positive integers n.
   (a) Determine the values of a, b, c, d, e.
   (b) Prove that the above displayed equality holds for all positive integers n.
   [Hint: mathematical induction.]