

MATH 314 ASSIGNMENT 4, FALL 2016

Due in class on Friday, February 12

Part 1. Read 3.1–3.5 of Hoffman–Kunze.

Part 2. Do and hand in the following problems in Hoffman–Kunze.

- 3.1, problems 9, 12
- 3.2, problems 7, 11
- 3.3, problem 5
- 3.4, problems 8, 10
- 3.5, problem 5

Part 3. (extra credit)

- A. Find an \mathbb{R} -linear transformation S from \mathbb{C} to $M_2(\mathbb{R})$, the \mathbb{R} -vector space of all 2×2 -matrices with entries in \mathbb{R} such that S has the same property (b) as in problem 5 of §3.3, i.e.

$$S(z_1 \cdot z_2) = S(z_1) S(z_2) \quad \text{for all } z_1, z_2 \in \mathbb{C},$$

and

$$\dim_{\mathbb{R}}(\text{Im}(T) \cap \text{Im}(S)) = 1,$$

where $T : \mathbb{C} \rightarrow M_2(\mathbb{R})$ is the linear transformation in problem 5 of §3.3. (Recall that $\text{Im}(S)$ is the image of the linear transformation S .)

- B. Does there exist a field F , a positive integer n , and two $n \times n$ matrices $A, B \in M_n(F)$ such that $AB - BA = I_n$? (Compare this with problem 5 of §3.5.)