

# MATH 314 ASSIGNMENT 5, FALL 2016

Due in class on Friday, February 19

Part 1. Read 3.5-4.3 of Hoffman–Kunze.

Part 2. Do and hand in the following problems in Hoffman–Kunze.

- 3.5, problems 11, 13
- 3.6, problems 1, 2
- 3.7, problems 3, 4
- 4.2, problems 7, 8

Part 3. (extra credit) For each natural number  $n$ , let  $P_n$  be the vector space over  $\mathbb{R}$  consisting of all polynomials with coefficient in  $\mathbb{R}$  of degree at most  $n$ . Let  $T_n : P_n \rightarrow P_n$  be the linear operator which sends every polynomial  $f(x) \in P_n$  to  $\frac{d^2f}{dx^2} - \frac{df}{dx} - 2f(x)$ .

- For  $n = 2, 3$ , find a non-zero polynomial  $g_n(Y)$  in one variable  $Y$  with coefficients in  $\mathbb{R}$  such that  $g_n(T_n)$  is equal to the zero operator on  $P_n$ .
- For general  $n \in \mathbb{N}$ , find a non-zero polynomial  $g_n(Y)$  in one variable  $Y$  with coefficients in  $\mathbb{R}$  such that  $g_n(T_n)$  is equal to the zero operator on  $P_n$ .
- What is the smallest possible degree of  $g_n(Y)$ ?