MATH 350 PRACTICE PROBLEMS FOR THE FIRST MIDTERM

OCTOBER, 2015

1. Suppose that a is an element of $\mathbb{Z}/101\mathbb{Z}$ such that $a^{513} = 1$. Is it possible that a is a primitive 100-th root of 1 in $\mathbb{Z}/101\mathbb{Z}$? Either give an example of a primitive element whose 513rd power is 1, or prove that there does not exist such an element.

2. 3. Determine whether the following statements are true or false.
(a) For prime numbers p, the Legendre symbol $\left(\frac{5}{p}\right)$ depends only on the congruence class of p modulo 5.
(b) For prime numbers p, the Legendre symbol $\left(\frac{11}{p}\right)$ depends only on the congruence class of p modulo 11.
(c) For non-zero natural numbers a, b which are relatively prime, the Jacobi symbol $\left(\frac{a}{b}\right)$ depends only on the congruence class of a modulo b. (d) For non-zero natural numbers a, b which are relatively prime, the Jacobi symbol $\left(\frac{a}{b}\right)$ depends only on the congruence class of b modulo $4a$.

3. Let p, q be prime numbers, $p \neq q$. Find a natural number n with $0 \neq n < pq$ such that $p^{2q-1} + q^{2p-1} \equiv n \pmod{pq}$. (The number n should be given in terms of p and q.)

4. Suppose that d is a positive odd integer such that the Jacobi symbol $\left(\frac{-1}{d}\right) = 1$. Is -1 necessarily a square in $\mathbb{Z}/d\mathbb{Z}$? Either give a proof, or provide a counter-example.

5. Prove or disprove the following statement: For every positive integer $n \geq 2$ there exists an element $(a \in \mathbb{Z}/n\mathbb{Z})^\times$ such that every element of $(\mathbb{Z}/n\mathbb{Z})^\times$ is a power of a?