MATH 350 ASSIGNMENT 3, FALL 2015

Due in class on Friday, September 18th

Read Chapters 10 to Chapter 13
Part 1. From the textbook *A friendly introduction to number theory*.

- Exercise 10.1 (b) and the following statement (a$'$): Show that $B^2 \equiv 1 \pmod{m}$.

  (Note that we only want to consider integers $m \geq 2$.)

- Exercise 11.12.
- Exercise 12.3 (a), (b)
- Exercise 13.6 (a), (b)

Part 2. Extra credit problems:

- Prove 10.1 (a) AND the pattern/statement in 10.1 (b) (which you found from a suitably large sample values of m) for general m.

  (The statement you prove should take the following form: $\prod_{a \in (\mathbb{Z}/m\mathbb{Z})^\times} = -1 \bmod m$ if and only if such and such a condition on m holds; otherwise $\prod_{a \in (\mathbb{Z}/m\mathbb{Z})^\times} = 1 \bmod m$.)

- Exercise 12.3 (c).

  [Hint: Don't be intimidated by the comment "This is quite difficult.". You might want to sum these $p-1$ terms in pairs, compute $\frac{1}{i} + \frac{1}{p-i}$ in $\mathbb{Z}/p^2\mathbb{Z}$ first.]

- Exercise 13.6 (c). In addition, determine by exact formula for the difference

$$\int_2^x \frac{dt}{\log t} - \left(\log(\log x) + \sum_{k=1}^{\infty} \frac{(\log x)^k}{k \cdot k!} \right),$$

  and also get a numerical estimate of this difference.

  [You should also show that the series converges.]