MATH 350 ASSIGNMENT 2, SPRING 2017

Due in class on Monday, January 30

Part 1. From the textbook *A friendly introduction to number theory*.

- Exercise 5.3
- Exercise 6.5
- Exercise 8.3
- Exercise 9.2
- Exercise 10.1 (a)

https://people.mpim-bonn.mpg.de/zagier/files/doi/10.2307/2323918/fulltext.pdf Part 2.

A. Define a function $d : \mathbb{N}_{\geq 1} \to \mathbb{N}$ by

$$d(n) = \sum_{d|n} 1$$

In other words $d(n)$ is the number of positive integers dividing n. Suppose you know the factorization of n: $n = p_1^{a_1} \cdots p_k^{a_k}$, where $p_1, \dots, p_k$ are mutually distinct prime numbers and $a_1, \dots, a_k$ are positive integers. What is $d(n)$?

Part 3. Extra credit problems.

- Exercise 6.6. Note that a proof of 6.6 (d) implies that if $\gcd(a,b) = 1$ for two positive integers a, b, there exists a natural number n_0 such that for every natural number $n \geq n_0$, there exist natural number x, y such that $n = ax + by$.
- Exercise 8.4
- Exercise 9.3

  https://people.mpim-bonn.mpg.de/zagier/files/doi/10.2307/2323918/fulltext.pdf
- Show that for every positive real number δ, there exists a natural number $n(\delta)$ such that $d(n) \leq n^{\delta}$ for all $n \geq n(\delta)$.

  [You can also prove partial results for some $\delta < 1$, such as $1/2$.]
- A famous theorem of Fermat asserts that every prime number p whichi is congruent to 1 modulo 4 is a sum of the square of two integers. Read the one-sentence proof of Don Zagier `https://people.mpim-bonn.mpg.de/zagier/files/doi/10.2307/2323918/fulltext.pdf`, and explain why that sentence is a proof. In other words, expand that sentence into a proof which is more easily understandable.