MATH 350 ASSIGNMENT 5, SPRING 2017

Due in class on Monday, February 20

Part 1. From the textbook *A friendly introduction to number theory*.

- Exercise 21.1 (a), (c) 4th edition (= Exercise 24.1 (a), (c) 3rd edition)

Part 2.

1. Find the number of solutions of the congruences

    (a) $x^2 \equiv 5 \pmod{73}$

    (b) $x^2 \equiv 3 \pmod{73}$

    (c) $x^2 \equiv 2 \pmod{110}$

    (d) $21x^2 \equiv 6 \pmod{51}$

Part 3. Extra credit problems.

A. (a) Show that for every positive integer n prime to 10, the decimal expansion the fraction $\frac{1}{n}$ is purely cyclic whose period is equal to the order of 10 mod n in $(\mathbb{Z}/n\mathbb{Z})^{\times}$.

    (b) Try to extend the statement (a) above to general fraction, i.e. determine the general form of the decimal expansion of a rational number.

B. Let p be an odd prime number.

    (a) Show that $x^{p-1} - 1 \equiv (x-1)(x-2)(x-3)\cdots(x-p+1) \pmod{p}$.

    (b) Determine the element
    $$\prod_{1 \le i \le p-1} i \bmod p$$
    of $\mathbb{Z}/p\mathbb{Z}$.

    (c) Determine the element
    $$\prod_{1 \le i < j \le p-1} i \cdot j \bmod p$$
    of $\mathbb{Z}/p\mathbb{Z}$.