# MATH 350 ASSIGNMENT 7, SPRING 2017

Due in class on Monday, March 13

Part 1. From the textbook *A friendly introduction to number theory*.

- Problem 29.1 (a), (c) of the 4th edition (= Problem 22.1 (a), (c) of the 3rd edition)
- Problem 29.3 (b) of the 4th edition (= Probme 22.3 (b) of the third edition)
- Problem 29.4 (a), (b), (c) of the 4th edition (= Problem 22.4 (a), (b), (c) of the third edition)

Part 2. Extra credit problems
A. Problem 29.3 (c) of the 4th edition (= Probme 22.3 (c) of the third edition)

B. Define a function $\lambda : \mathbb{N}_{>0} \to \{1, -1\}$ as follows: if $n = p_1^{e_1} \cdots p_r^{e_r}$ is the factorization of $n$ into a product of prime numbers, then $\lambda(n) = (-1)^{e_1+\cdots+e_r}$. Notice that $\lambda(1) = 1$, corresponding to the case when $r = 1$. Prove the following statements.

(a) $\lambda(mn) = \lambda(m) \cdot \lambda(n)$ for all positive integers $m, n$.

(b) $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square, i.e. } n = m^2 \text{ for an integer } m \\ 0 & \text{if } n \text{ is not a square} \end{cases}$

(c) $\sum_{n=1}^{m} \lambda(n) \lfloor m/n \rfloor = \lfloor \sqrt{m} \rfloor$ for every positive integer $m$.

[Hint: Write the sum in question as a double sum, interchange the order of summation and use (b).]

(d) $\left| \sum_{n=1}^{m} \frac{\lambda(n)}{n} \right| \leq 2$ for every positive integer $m$.

[Hint: Use (c).]