

MATH 350 ASSIGNMENT 8, SPRING 2017

Due in class on Monday, March 20

Part 1.

1. For $p = 5, 7$, find elements a, b in $(\mathbb{Z}/p^3\mathbb{Z})^\times$ such that the order of a is $p - 1$ and the order of b is $(p - 1)p^2$.
2. Show for any positive integer $n \geq 3$, the maximal possible order of elements of $(\mathbb{Z}/2^n\mathbb{Z})^\times$ is 2^{n-2} .

Part 2. From the textbook *A friendly introduction to number theory*.

- Exercise 13.3
- Exercise 13.5

Part 3. Extra credit problem:

- A. Exercise 13.6 of *A friendly introduction to number theory*.
- B. Show that there are infinitely many prime number p which are congruent to 1 modulo 4. (This is proved later in the book. The challenge here is to find a proof on your own. If you do chances are that it is different from the one given in the book.)