1. Do Gallier/Shatz problem 93.

2. Do Gallier/Shatz problem 96, parts 1, 2, 3, 5.

3. Let $R$ be a commutative ring and let $M$ be an $R$-module. Let $I$ be a maximal member (with respect to inclusion) in the family

$$\{\text{ann}_R(m) \mid 0 \neq m \in M\}$$

of annihilators of non-zero elements of $M$. Show that $I$ is a prime ideal.

4. Let $R$ be a commutative ring. Let $f_1, \ldots, f_n$ be elements of $R$ such that none of the $f_i$’s is nilpotent and the ideal generated by $f_1, \ldots, f_n$ is equal to $R$. Let $M_i$ be an $R_{f_i}$-module for each $i = 1, \ldots, n$. Suppose that we have isomorphisms $\alpha_{ij} : (M_j)_{f_i} \xrightarrow{\sim} (M_i)_{f_j}$ of $R_{f_i}f_j$-modules for all pairs $(i, j)$ such that the equality

$$\left(\alpha_{ij}\right)_{f_k} \circ \left(\alpha_{jk}\right)_{f_i} = \left(\alpha_{ik}\right)_{f_j}$$

holds for all triples $i, j, k$. Here $(\alpha_{ij})_{f_k} : (M_j)_{f_i f_k} \xrightarrow{\sim} (M_i)_{f_j f_k}$ is the localization of $\alpha_{ij}$ with respect to the multiplicative closed subset $\{f_k^N\}$. Show that there exists an $R$-module $M$ and isomorphisms

$$\beta_i : M_{f_i} \xrightarrow{\sim} M_i$$

such that

$$\alpha_{ij} \circ (\beta_j)_{f_i} = (\beta_i)_{f_j}$$

for all $i, j$.

5. Let $R$ be a commutative ring satisfying the following properties.

- The localization $R_m$ is noetherian for every maximal ideal $m$ of $R$.
- For every non-zero element $f \in R$, there exist only a finite number of maximal ideals of $R$ which contains $f$.

Show that $R$ is noetherian.