1. Compute the character table of the quaternion group with 8 elements.

2. Let \( N \) be the subgroup of \( \text{GL}_3(\mathbb{F}_3) \) consisting of all upper-triangular unipotent \( 3 \times 3 \) matrices with entries in \( \mathbb{F}_3 \). Determine the character table of \( N \).

3. Determine the character table of the quaternion group \( Q_{16} \) with 16 elements defined in Assignment 2.

4. Let \( G \) be a finite group whose cardinality is an odd number. Prove that \( \chi(i) \neq \overline{\chi(i)} \) for any nontrivial complex irreducible character \( \chi(i) \). (Hint: Use a suitable orthogonality relation to show that if \( \chi(i) = \overline{\chi(i)} \), then \( \frac{\chi(i)(1)}{2} \) would be an algebraic integer.)

5. Let \( G \) be a finite group and let \( \chi(i) \) be the character of an irreducible linear representation of \( G \) on a finite dimensional vector space \( V \) over \( \mathbb{C} \). Let \( n = \text{Card}(G) \).
   
   (i) Show that \( \chi(\sigma) \) is an algebraic integer in the cyclotomic field \( \mathbb{Q}(\mu_n) \) for every \( \sigma \in G \).
   
   (ii) Let \( \sigma \) be an element of \( G \) such that \( \chi(\sigma) \neq 0 \). Show that
   
   \[
   \text{Tr}_{\mathbb{Q}(\mu_n)/\mathbb{Q}}(\chi(\sigma) \cdot \overline{\chi(\sigma)}) \geq [\mathbb{Q}(\mu_n) : \mathbb{Q}].
   \]
   
   (Hint: Use the fact that the arithmetic means of a finite number of elements in \( \mathbb{R}_{\geq 0} \) is bigger than or equal to their geometric means.)
   
   (iii) Assume that \( \dim_{\mathbb{C}}(V) > 1 \). Prove that there exists an element \( \sigma \in G \) such that \( \chi(\sigma) = 0 \).
   
   (iv) Let \( \tau \) be an automorphism of the cyclotomic field \( \mathbb{Q}(\mu_n) \). Show that there exists an integer \( a \) such that \( \tau(\chi(\sigma)) = \chi(\sigma^a) \) for every \( \sigma \in G \).

6. (extra credit) Determine the character table of the finite group \( \text{SL}_2(\mathbb{F}_5) \).