1. (Hatcher, number 2.1.14) Determine whether there exists a short exact sequence
\[ 0 \to \mathbb{Z}_4 \to \mathbb{Z}_8 \oplus \mathbb{Z}_2 \to \mathbb{Z}_4 \to 0. \]

More generally, determine which abelian groups \( A \) fit into a short exact sequence
\[ 0 \to \mathbb{Z}_{p^m} \to A \to \mathbb{Z}_{p^n} \to 0 \]
with \( p \) prime. What about the case of short exact sequences
\[ 0 \to \mathbb{Z} \to A \to \mathbb{Z}_n \to 0? \]

2. (Hatcher, number 2.1.20) Show that \( \tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX) \) for all \( n \), where \( SX \) is the suspension of \( X \); \( SX = X \times I/(x, 1) \sim (y, 1) \) and \( (x, 0) \sim (y, 0) \) for all \( x, y \in X \).

3. (Hatcher, number 2.1.26) Show that \( H_1(X, A) \) is not isomorphic to \( \tilde{H}_1(X/A) \) if \( X = [0, 1] \) and \( A \) is the sequence \( 1, \frac{1}{2}, \frac{1}{3}, \ldots \) together with its limit 0.

4. (Hatcher, number 2.2.2) Given a map \( f : S^{2n} \to S^{2n} \), show that there is some point \( x \in S^{2n} \) with either \( f(x) = x \) or \( f(x) = -x \). Deduce that every map \( \mathbb{RP}^{2n} \to \mathbb{RP}^{2n} \) has a fixed point. Construct maps \( \mathbb{RP}^{2n-1} \to \mathbb{RP}^{2n-1} \) without fixed points from linear transformations \( R^{2n} \to R^{2n} \) without eigenvectors.

5. (Hatcher, number 2.2.8) A polynomial \( f(z) \) with complex coefficients, viewed as a map \( \mathbb{C} \to \mathbb{C} \), can always be extended to a continuous map of one-point compactifications \( \hat{f} : S^2 \to S^2 \). Show that the degree of \( \hat{f} \) equals the degree of \( f \) as a polynomial. Show also that the local degree of \( \hat{f} \) at a root of \( f \) is the multiplicity of the root.

6. Prove the Brouwer fixed point theorem: Any continuous map \( f : D^n \to D^n \) admits a fixed point.

7. By considering \( T^2 \) as \( \mathbb{R}^2/\mathbb{Z}^2 \), we see that \( SL(2, \mathbb{Z}) \) acts on \( T^2 \) via homeomorphisms. For \( \phi \in SL(2, \mathbb{Z}) \), let \( Y_\phi = (S^1 \times D^2) \cup_\phi (S^1 \times D^2) \). Determine \( H_*(Y_\phi) \) in terms of \( \phi \).

8. Given \( \phi \in SL(2, \mathbb{Z}) \), consider the mapping torus of \( T^2 \) with respect to \( \phi \):
\[ X_\phi = T^2 \times I/(x, 1) \sim (\phi(x), 0). \]
Compute \( H_*(X_\phi) \) in terms of \( \phi \).

9. For any knot in \( \mathbb{R}^3 \), compute \( H_*(\mathbb{R}^3\setminus \nu K) \), where \( \nu K \) is a neighborhood of the knot.