Expected Value - Revisited

▶ An experiment is a **Bernoulli Trial** if:
  ▶ there are two outcomes (success and failure),
  ▶ the probability of success, $p$, is always the same,
  ▶ the trials are independent.

▶ The probability of failure is $1 - p$.

▶ Suppose we repeat a Bernoulli trial $n$ times.
  ▶ How many successes do we expect to get? (what is the expected value, $\mu$?)
  ▶ How much variance is there $(\sigma^2)$, in the expected number of successes?
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Flipping a Coin

- Toss a coin 6 times, and count the number of heads.
Flipping a Coin

- Toss a coin 6 times, and count the number of heads.
- We are repeating a Bernoulli trial 6 times.
Toss a coin 6 times, and count the number of heads.

We are repeating a Bernoulli trial 6 times.

<table>
<thead>
<tr>
<th># of heads</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td><img src="http://example.com/probability.png" alt="image" /></td>
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The expected value is:

$$\mu = \frac{1}{64} \cdot 0 + \frac{6}{64} \cdot 1 + \frac{15}{64} \cdot 2 + \frac{20}{64} \cdot 3 + \frac{15}{64} \cdot 4 + \frac{6}{64} \cdot 5 + \frac{1}{64} \cdot 6 = 3.$$

The variance is:

$$\sigma^2 = (0 - 3)^2 \cdot \frac{1}{64} + (1 - 3)^2 \cdot \frac{6}{64} + \cdots + (6 - 1)^2 \cdot \frac{1}{64} = 3.$$
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<tr>
<td>probability</td>
<td>1/64</td>
<td>6/64</td>
<td>15/64</td>
<td>20/64</td>
<td>15/64</td>
<td>6/64</td>
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The expected value is:

\[ \mu = 0 \cdot \frac{1}{64} + 1 \cdot \frac{6}{64} + \cdots + 6 \cdot \frac{1}{64} = 3. \]
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$$
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$$

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$$
\sigma^2 = (0 - 3)^2 \cdot \frac{1}{64} + (1 - 3)^2 \cdot \frac{1}{64} + \cdots + (6 - 1)^2 \cdot \frac{1}{64} = \frac{3}{2}.
$$
Expected Value and Variance

- We want better formulas.

In $n$ Bernoulli trials with success probability $p$, we have:

\[ \mu = np \]

\[ \sigma^2 = np(1-p) \]
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We can view this as a Bernoulli trial, by looking at each star.

- $n$ is 300 billion.
- Want $p$. 
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The Drake Equation is roughly:

\[ P = p_{\text{planet}} \cdot p_{\text{life}} \cdot p_{\text{intelligence}} \cdot p_{\text{civilization}}. \]
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- Estimates are:
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\[ \mu \approx 7.8 \text{ billion} \]
The Drake Equation

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So $p = .026$.

So $\mu$ is approximately 7.8 billion.
Complaints?

- Criticisms:

Civilizations don't last forever (need more complicated equation).

Multiplying probabilities, we don't really know \( p_{\text{life}} \), \( p_{\text{intelligence}} \), and \( p_{\text{civilization}} \).
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Flipping a Coin

- Going back to flipping a coin 6 times.
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- Plot the probabilities of getting $k$ heads,
Going back to flipping a coin 6 times.

Plot the probabilities of getting \( k \) heads, and

\[
\frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]
Flipping a Coin
Flipping a Coin

- Now flip a coin 20 times.
Flipping a Coin

- Now flip a coin 20 times.
  - What is $\mu$?
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- Now flip a coin 20 times.
  - What is $\mu$?
  - What is $\sigma^2$?
Now flip a coin 20 times.

- What is $\mu$?
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Where is the curve centered at?

The standard deviation/variance measures how wide the curve is.
Flipping a Coin

- Moral: when $n$ gets large, the distribution of the number of successes looks like a bell-shaped curve.
- Where is the curve centered at?
- The standard deviation/variance measures how wide the curve is.
- The area under the curve is always 1.
If a certain variable is distributed as a bell-shaped curve, we say that the variable follows a normal distribution.
Normal Distributions

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- Examples: Bernoulli trials, heights of people, IQ scores, light bulb lifetimes...
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Normal Distributions

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- Examples: Bernoulli trials, heights of people, IQ scores, light bulb lifetimes...
- We need to know 2 numbers to describe the normal distribution:
  - $\mu$: the mean, where the curve is centered.
  - $\sigma$: the standard deviation, which specifies how spread out the bell is.