Recall the Candidate-Voter Model:

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- Win by random draw if candidates tie
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  - Cost of 100 to run
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- So a Nash equilibrium occurs when:
  - All candidates who run tie
  - No one can opt to run and tie or win
The Candidate-Voter Model

- Properties of this model:
  - There are many Nash equilibria
  - Not all equilibria have candidates crowded at the median
  - If candidates become too extreme, more central candidates will jump in
  - If you enter on the left, you make it more likely that someone on the right wins (splitting the vote)

- Problems?
  - Everyone decides whether or not to run at once
  - Not everyone can practically run
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What's the name of this game?

Rock Paper Scissors

Are there any Nash equilibria?

No

What is the best strategy?

Should be to pick each of rock, paper, and scissors randomly with probability of \(\frac{1}{3}\) (Denote this as \(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\)).

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What is the expected payout of \((1/3, 1/3, 1/3)\) against \((1, 0, 0)\)?

\(u((1/3, 1/3, 1/3), (1, 0, 0)))\)
Expected Payout

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- What is the expected payout of \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \) against \( (1, 0, 0) \)?
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\[ u((\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (1, 0, 0)) \]

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Note that the expected payout is weighted average of the payouts of the pure strategies (with positive probabilities)
Weighted Averages

- How can you raise the average batting average of a baseball team?
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  - By cutting people with a low batting average
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- How can you raise the average batting average of a baseball team?
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  - If the average batting average is maximized, all players must have the same batting average
- If $p_i$ is a best response to the other strategies, all the pure strategies used in $p_i$ are best responses to $p_{-i}$
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<tr>
<td><strong>C</strong></td>
<td><strong>D</strong></td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>3, 2</td>
</tr>
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You

Is $\left(\frac{1}{2}, \frac{1}{2}\right)$ a best response to $\left(0, 1\right)$?

- No - you should drop C
### Weighted Averages

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- Note that pure Nash equilibria are still Nash equilibria