Evolutionarily Stable Strategies

Idea:

- If \( s \) is an **evolutionarily stable strategy**, any other strategy \( s^* \) will die off when competing against mixed population.

\[
\text{If } u(s, s) > u(s^*, s) \quad \text{then } s \text{ is stable.}
\]

\[
\text{If } u(s^*, s) > u(s, s) \quad \text{then } s \text{ is not stable.}
\]

\[
\text{If } u(s, s) = u(s^*, s^*) \quad \text{we need to look at } (1 - \epsilon)u(s, s) + \epsilon u(s^*, s^*) > (1 - \epsilon)u(s^*, s^*) + \epsilon u(s^*, s^*)
\]

\[
\text{Then } s \text{ will be evolutionarily stable only if } u(s, s^*) > u(s^*, s^*).
\]

\[
\text{If } s \text{ is evolutionarily stable, } (s, s^*) \text{ is a Nash equilibrium.}
\]

\[
\text{If } (s, s^*) \text{ is a Nash equilibrium, } s \text{ is not necessarily evolutionarily stable.}
\]
Evolutionarily Stable Strategies

Idea:

- If $s$ is an evolutionarily stable strategy, any other strategy $s^*$ will die off when competing against mixed population.
- Population is mostly $s$.
Evolutionarily Stable Strategies

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$$\left(1 - \epsilon\right) u(s, s) + \epsilon u(s, s^*) > \left(1 - \epsilon\right) u(s^*, s) + \epsilon u(s^*, s^*)$$
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- If $s$ is evolutionarily stable, $(s, s)$ is a Nash equilibrium
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Another definition for evolutionarily stable strategies:
In a 2-player symmetric game, a strategy \( s \) is **evolutionarily stable** if:

1. \((s, s)\) is a Nash equilibrium, and
2. If \( u(s, s) = u(s^*, s) \) then \( u(s, s^*) > u(s^*, s^*) \)
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- This definition is far easier to check
If \( s \) is evolutionarily stable, is \((s, s)\) a Nash equilibrium?

- Yes

If \((s, s)\) is a Nash equilibrium, is \( s \) evolutionarily stable?

- Not necessarily: if \( u(s, s^*) = u(s^*, s^*) \), need to know that 
  \[ u(s, s^*) > u(s^*, s^*) \]

If \( s \) is evolutionarily stable, is it possible that \( s^* \) strongly dominates \( s \)?

- No

If \( s^* \) strictly dominates \( s \), it will do better against \( s \)(and \((s, s)\) is not a Nash equilibrium)

If \( s \) is evolutionarily stable, is it possible that \( s^* \) is not strongly dominated by \( s \)?

- Yes
Handout #7

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Evolutionarily Stable Strategies

What pure symmetric Nash equilibria are there?

_What will happen to the population?
- It will be mixed

_The strategy \( p = \left( \frac{2}{3}, \frac{1}{3} \right) \) gives a symmetric Nash equilibrium

_Will it do strictly better against itself than any other strategy?
- No - because it is a mixed strategy

_Need to check how \( p \) does against any other mixed strategy (vs. how that strategy does against itself)

_\( p \) is a mixed evolutionarily stable strategy
Evolutionarily Stable Strategies

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Mixed Evolutionarily Stable Strategies

Can mixed evolutionarily stable strategies happen in nature?
Mixed Evolutionarily Stable Strategies

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Common side-blotched lizard
Mixed Evolutionarily Stable Strategies

Can mixed evolutionarily stable strategies happen in nature?

Common side-blotched lizard
- Males have three possible colorings (orange-blue-yellow)
Mixed Evolutionarily Stable Strategies

Can mixed evolutionarily stable strategies happen in nature?

Common side-blotched lizard
- Males have three possible colorings (orange-blue-yellow)
- Colorings corresponding to mating habits
Common Side-Blotched Lizard

- Blue lizards (dominant) guard small territory and have a single mate
Common Side-Blotched Lizard

- Blue lizards (dominant) guard small territory and have a single mate
- Orange lizards (ultradominant) have larger territory, and try to claim all females in the territory

Only evolutionarily stable strategy is Orange
Common Side-Blotched Lizard

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- If there were just these two types, what would happen?
Common Side-Blotched Lizard

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\[
\begin{array}{cc}
\text{Orange} & \text{Blue} \\
1, 1 & V, 0 \\
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- Yellow lizards (sneakers) look similar to females
Common Side-Blotched Lizard

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Common Side-Blotched Lizard

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No pure evolutionarily stable strategies

\((1, 1, 1)\) is evolutionarily stable
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- No pure evolutionarily stable strategies
- \((\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\) is evolutionarily stable