MATH 104 – Practice Problems for Final Exam

1. Find the area between:
   (a) $y = 3\sin(\pi x)$ and $y = 6x$ (the part for $x > 0$).
   (b) $y = \dfrac{1}{\sqrt{x^2+4}}$ and the x axis for $0 \le x \le 3$
   (c) $y = x^2$ and $x = y^2$

2. Calculate the volume obtained by rotating:
   (a) The region in problem 1a around the x-axis
   (b) The region in problem 1a around the y-axis
   (c) The region in problem 1b around the x-axis
   (d) The region in problem 1b around the y-axis
   (e) The region in problem 1c around the x-axis
   (f) The region in problem 1c around the y-axis
   (g) The region in problem 1c around the line $x = 1$
   (h) The region in problem 1c around the line $y = -1$

3. Calculate the length:
   (a) of the part of $y = \sqrt{x}$ from $x = 0$ to $x = 1$.
   (b) of the part of $y = x^{3/2}$ from $x = 1$ to $x = 6$
   (c) of the part of $y = \ln(\sin x)$ for $0 \le x \le \pi/4$ (careful!).
   (d) What about the preceding curve for $\pi/6 \le x \le \pi/4$?
   (e) of the part of $y = e^x$ for $0 \le x \le 1$

4. Calculate the surface area obtained by rotating:
   (a) The region in problem 3a around the x-axis
   (b) The region in problem 3a around the y-axis
   (c) The region in problem 3b around the x-axis
   (d) The region in problem 3b around the y-axis.
   (e) The region in problem 3e around the x-axis

5. Integrate: (straightforward)
   (a) $\int x^4 \ln(2x)\, dx$
   (b) $\int x^2 \cos(3x)\, dx$
   (c) $\int \frac{x+2}{x^2+8x+15}\, dx$
   (d) $\int \sqrt{1-4x^2}\, dx$
   (e) $\int \frac{1}{\sqrt{x}-1}\, dx$
   (f) $\int \frac{\sin^2(\ln x)}{x}\, dx$
   (g) $\int \frac{\sec^2(\ln x)}{x\sqrt{1-\tan(\ln x)}}\, dx$

6. Integrate: (trickier)
   (a) $\int \cos^4(2x)\, dx$
   (b) $\int \frac{\sqrt{x^2+16}}{x}\, dx$
   (c) $\int \frac{e^t}{e^{2t}+4}\, dt$
   (d) $\int \sqrt{1-e^{2x}}\, dx$
   (e) $\int \cos\sqrt{x}\, dx$

7. Evaluate:
   (a) $\int_e^\infty \frac{1}{x(\ln x)^2}\, dx$
   (b) $\int_0^\infty \frac{dx}{(x+1)(x+4)}$
   (c) $\int_0^1 \sqrt{\frac{1-y}{y}}\, dy$

8. For a certain value of k, each of the following functions is a probability distribution. Find the value of k, and then find the mean and median of the distribution. Also find the probablity that $x > 1$ for each distribution.

(a) $f(x) = 3e^{kx}$ for $x > 0$ (zero otherwise)

(b) $f(x) = \dfrac{k}{1+x^2}$ for all x.

(c) $f(x) = k\ln(2+x)$ for $0 < x < 2$ (zero otherwise, you'll have to settle for an approximate value for the median).

9. Solve the initial-value problem:

   (a) $y' + y = 2$, $y(0) = 1$

   (b) $\dfrac{dy}{dx} = 1 - y + x^2 - yx^2$, $y(0) = 0$.

   (c) $y' - xy = x$, $y(0) = 3$.

10. Suppose the rate of change of a quantity y is proportional to ******, and $y(0) = 1$ and $y(1) = 2$, then what is y?

    (a) ******* is y itself

    (b) ******* is y^2

    (c) ******* is $\sqrt{y}$

    (d) ******* is $y + 1$.

11. Find the limit of the sequence:

    (a) $\left\{\dfrac{\ln(1+3/n)}{\sin(2/n)}\right\}$

    (b) $\left\{\left(\dfrac{n+1}{n+2}\right)^n\right\}$

    (c) $\left\{(3^n + 5^n)^{1/n}\right\}$

12. Which series converge? (straightforward)

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{1+n^4}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{1+n^5}$

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{1+e^{-n}}$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

13. If you put $(-1)^n$ into each of the series in the preceding problem, which ones converge absolutely, converge conditionally, or diverge?

14. Which series converge (trickier)

    (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n)^2)}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n!)}{n^4+2n+1}$

    (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\tan(1/n)}{\ln n}$

    (d) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{e^n}{n!}}$

15. If you put $(-1)^n$ into each of the series in the preceding problem, which ones converge absolutely, converge conditionally, or diverge?

16. For which x do the following series converge?

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{1+n^2}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{ne^n}$

    (d) $\displaystyle\sum_{n=1}^{\infty} n^2x^n$

17. To what functions to the series in parts a, c and d of the preceding problem converge?

18. Find the Taylor series for

    (a) e^{-x^2} centered at 0

    (b) $\displaystyle\int_0^x \cos t^3\, dt$ centered at $x=0$

    (c) $\sqrt{x}$ centered at $x=1$

    (d) the solution of $y' - xy = 0$, $y(0) = 1$ centered at $x = 0$

19. Estimate to the nearest 0.001 (with explanation):

   (a) $\int_0^{0.2} \cos\sqrt{x}\,dx$

   (b) $\sqrt{3.9}$ (use the series for $\sqrt{x}$ around $x = 4$).

   (c) $e^{-0.2}$

20. If $f(x) = x^3 \cos x^2$, then what is $f^{(13)}(0)$? (this is the value of the thirteenth derivative of f at 0).