1. Find the area between:
   (a) \( y = 3 \sin(\pi x) \) and \( y = 6x \) (the part for \( x > 0 \)).
   (b) \( y = \frac{1}{\sqrt{x^2 + 4}} \) and the \( x \) axis for \( 0 \leq x \leq 3 \)
   (c) \( y = x^2 \) and \( x = y^2 \)

2. Calculate the volume obtained by rotating:
   (a) The region in problem 1a around the \( x \)-axis
   (b) The region in problem 1a around the \( y \)-axis
   (c) The region in problem 1b around the \( x \)-axis
   (d) The region in problem 1b around the \( y \)-axis
   (e) The region in problem 1c around the \( x \)-axis
   (f) The region in problem 1c around the \( y \)-axis
   (g) The region in problem 1c around the line \( x = 1 \)
   (h) The region in problem 1c around the line \( y = -1 \)

3. Calculate the length:
   (a) of the part of \( y = \sqrt{x} \) from \( x = 0 \) to \( x = 1 \).
   (b) of the part of \( y = x^{3/2} \) from \( x = 1 \) to \( x = 6 \)
   (c) of the part of \( y = \ln(\sin x) \) for \( 0 \leq x \leq \pi/4 \) (careful!).
   (d) What about the preceding curve for \( \pi/6 \leq x \leq \pi/4 \) ?
   (e) of the part of \( y = e^x \) for \( 0 \leq x \leq 1 \)

4. Calculate the surface area obtained by rotating:
   (a) The region in problem 3a around the \( x \)-axis
   (b) The region in problem 3a around the \( y \)-axis
   (c) The region in problem 3b around the \( x \)-axis
   (d) The region in problem 3b around the \( y \)-axis.
   (e) The region in problem 3e around the \( x \)-axis
5. Integrate: (straightforward)
   (a) \( \int x^4 \ln(2x) \, dx \)
   (b) \( \int x^2 \cos(3x) \, dx \)
   (c) \( \int \frac{x + 2}{x^2 + 8x + 15} \, dx \)
   (d) \( \int \sqrt{1 - 4x^2} \, dx \)
   (e) \( \int \frac{1}{\sqrt{x - 1}} \, dx \)
   (f) \( \int \frac{\sin^2(\ln x)}{x} \, dx \)
   (g) \( \int \frac{\sec^2(\ln x)}{x \sqrt{1 - \tan(\ln x)}} \, dx \)

6. Integrate: (trickier)
   (a) \( \int \cos^4(2x) \, dx \)
   (b) \( \int \frac{\sqrt{x^2 + 16}}{x} \, dx \)
   (c) \( \int \frac{e^t}{e^{2t} + 4} \, dt \)
   (d) \( \int \sqrt{1 - e^{2x}} \, dx \)
   (e) \( \int \cos \sqrt{x} \, dx \)

7. Evaluate:
   (a) \( \int_0^\infty \frac{1}{x(ln x)^2} \, dx \)
   (b) \( \int_0^{\infty} \frac{dx}{(x + 1)(x + 4)} \)
   (c) \( \int_0^1 \frac{\sqrt{1 - y}}{y} \, dy \)

8. For a certain value of \( k \), each of the following functions is a probability distribution. Find the value of \( k \), and then find the mean and median of the distribution. Also find the probability that \( x > 1 \) for each distribution.
(a) \( f(x) = 3e^{kx} \) for \( x > 0 \) (zero otherwise)
(b) \( f(x) = \frac{k}{1 + x^2} \) for all \( x \).
(c) \( f(x) = k \ln(2 + x) \) for \( 0 < x < 2 \) (zero otherwise, you’ll have to settle for an approximate value for the median).

9. Solve the initial-value problem:
(a) \( y' + y = 2, \ y(0) = 1 \)
(b) \( \frac{dy}{dx} = 1 - y + x^2 - yx^2, \ y(0) = 0 \).
(c) \( y' - xy = x, \ y(0) = 3 \).

10. Suppose the rate of change of a quantity \( y \) is proportional to \( \bullet \bullet \bullet \bullet \bullet \), and \( y(0) = 1 \) and \( y(1) = 2 \), then what is \( y \)?
(a) \( \bullet \bullet \bullet \bullet \bullet \) is \( y \) itself
(b) \( \bullet \bullet \bullet \bullet \bullet \) is \( y^2 \)
(c) \( \bullet \bullet \bullet \bullet \bullet \) is \( \sqrt{y} \)
(d) \( \bullet \bullet \bullet \bullet \bullet \) is \( y + 1 \).

11. Find the limit of the sequence:
(a) \( \left\{ \frac{\ln(1 + 3/n)}{\sin(2/n)} \right\} \)
(b) \( \left\{ \left( \frac{n + 1}{n + 2} \right)^n \right\} \)
(c) \( \left\{ (3^n + 5^n)^{1/n} \right\} \)

12. Which series converge? (straightforward)
(a) \( \sum_{n=1}^{\infty} \frac{n^3}{3^n} \)
(b) \( \sum_{n=1}^{\infty} \frac{n^3}{1 + n^4} \)
(c) \( \sum_{n=1}^{\infty} \frac{n^3}{1 + n^5} \)
(d) \( \sum_{n=1}^{\infty} \frac{\cos^2 n}{1 + e^{-n}} \)
(e) \( \sum_{n=1}^{\infty} \frac{e^n}{n!} \)
13. If you put \((-1)^n\) into each of the series in the preceding problem, which ones converge absolutely, converge conditionally, or diverge?

14. Which series converge (trickier)
   (a) \(\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^2}\)
   (b) \(\sum_{n=1}^{\infty} \frac{\ln(n!)}{n^4 + 2n + 1}\)
   (c) \(\sum_{n=2}^{\infty} \frac{\tan(1/n)}{\ln n}\)
   (d) \(\sum_{n=1}^{\infty} \sqrt{\frac{e^n}{n!}}\)

15. If you put \((-1)^n\) into each of the series in the preceding problem, which ones converge absolutely, converge conditionally, or diverge?

16. For which \(x\) do the following series converge?
   (a) \(\sum_{n=1}^{\infty} \frac{(x + 2)^n}{n}\)
   (b) \(\sum_{n=1}^{\infty} \frac{(x - 3)^n}{1 + n^2}\)
   (c) \(\sum_{n=1}^{\infty} \frac{(-1)^n(x - 1)^n}{ne^n}\)
   (d) \(\sum_{n=1}^{\infty} n^2x^n\)

17. To what functions to the series in parts a, c and d of the preceding problem converge?

18. Find the Taylor series for
   (a) \(e^{-x^2}\) centered at 0
   (b) \(\int_{0}^{x} \cos t^3 \, dt\) centered at \(x = 0\)
   (c) \(\sqrt{x}\) centered at \(x = 1\)
   (d) the solution of \(y' - xy = 0, y(0) = 1\) centered at \(x = 0\)
19. Estimate to the nearest 0.001 (with explanation):

(a) \( \int_0^{0.2} \cos \sqrt{x} \, dx \)

(b) \( \sqrt{3.9} \) (use the series for \( \sqrt{x} \) around \( x = 4 \)).

(c) \( e^{-0.2} \)

20. If \( f(x) = x^3 \cos x^2 \), then what is \( f^{(13)}(0) \)? (this is the value of the thirteenth derivative of \( f \) at 0).