1. \( \int_{e}^{e^4} \frac{1}{x\sqrt{\ln x}} \, dx = \)

(a) 0  
(b) 2  
(c) 4  
(d) \( e^2 \)  
(e) \( e^4 - e \)

2. We know that the graph of a function \( \varphi(x) \) passes through the point (4,6). We also know that \(-1 \leq \varphi'(x) \leq 3\) for all \( x \) in the interval \([0, 10]\). What are the minimum and maximum values possible values for \( \varphi(7) \)?

(a) min = 7, max = 16  
(b) min = 4, max = 7  
(c) min = -1, max = 4  
(d) min = 6, max = 10  
(e) min = 4, max = 13  
(f) min = 3, max = 15

3. \( \int_{1}^{e} x\ln x \, dx = \)

(a) \( \frac{e^2}{4} - \frac{4}{e^2} \)  
(b) \( \frac{e^2}{2} - \frac{2}{e^2} \)  
(c) \( \frac{3e^2}{4} \)  
(d) \( \frac{e^2}{4} + \frac{1}{4} \)  
(e) \( \frac{3e^2}{2} - \frac{1}{4e^2} \)  
(f) \( \frac{e^2}{4} \)

4. What is the volume of the solid obtained by rotating the part of the graph of \( y = \cos x\sqrt{\sin x} \) between \( x = 0 \) and \( x = \frac{\pi}{2} \) around the \( x \)-axis?

(a) \( \pi \)  
(b) \( \frac{\pi}{2} \)  
(c) \( \frac{\pi}{3} \)  
(d) \( \frac{\pi}{4} \)  
(e) \( \frac{\pi}{6} \)

5. \( \lim_{b \to \infty} \int_{0}^{b} \arctan \frac{x}{1 + x^2} \, dx = \)

(a) \( \frac{\pi^2}{2} \)  
(b) \( \pi^2 \)  
(c) \( \frac{\pi^2}{4} \)  
(d) \( \frac{\pi^2}{8} \)  
(e) \( +\infty \)

6. The area between a curve \( y = f(x) \) and the \( x \)-axis between \( x = 1 \) and \( x = b \) is rotated around the \( x \) axis and the volume of the resulting solid is \( \frac{\pi}{3} (\ln b)^3 \), for all \( b > 1 \). What is \( f(x) \)?

(a) \( (\ln x)^2 \)  
(b) \( \frac{\ln x}{\sqrt{x}} \)  
(c) \( \frac{\ln x}{x} \)  
(d) \( \frac{(\ln x)^2}{x} \)  
(e) \( \frac{(\ln x)^2}{\sqrt{x}} \)
7. \[ \int_{0}^{\pi} \sin^2(2x) \, dx = \]
(a) 0 (b) 1 (c) 1/2 (d) \[ \pi/2 \] (e) \[ \pi \] (f) 2\[ \pi \]

8. Suppose that \( f(x) \) and \( g(x) \) are two differentiable functions, and we know the following values:
\[
f(0) = 2, \quad f(1) = 3, \quad g(0) = 1, \quad g(1) = 4.
\]
Then \[
\int_{0}^{1} \left( f(x)^2 g'(x) + 2f(x)f'(x)g(x) \right) \, dx =
\]
(a) 6 (b) 12 (c) 14 (d) 16 (e) 32 (f) 48

9. The area bounded by the curves \( y = x^2 - 1 \) and \( y = 2x + 7 \) is
(a) 4 (b) 9 (c) 16 (d) 24 (e) 30 (f) 36

10. If \( f(x) \) is continuous for all \( x \) and \( \lim_{x \to 0} \frac{f(x)}{x} = 4 \), then
\[
\begin{array}{ll}
(a) f(0) = 1 \text{ and } f'(0) = 4 & \text{ (b) } f(0) = 0 \text{ and } f'(0) = 4 \\
(c) f(0) = 4 \text{ and } f'(0) = 0 & \text{ (d) } f(0) = 0 \text{ and } f'(0) = 1 \\
(e) f(0) = 0 \text{ and } f'(0) = 0 & \text{ (f) } f(0) = 4 \text{ and } f'(0) = 1
\end{array}
\]

11. \[ \frac{d}{dx} \int_{0}^{\sin x} e^{t^2} \, dt = \]
(a) \( e^{\sin^2 x} \) (b) \( \cos x e^{\sin^2 x} \) (c) \( 2 \sin x \cos x e^{\sin^2 x} \) (d) \( \cos x + e^{\sin^2 x} \) (e) \( 2 \sin x e^{\sin^2 x} + e^{\sin^2 x} \) (f) \( e^{\cos^2 x} \)

12. \[ \int_{1/e}^{e} \frac{(\ln x)^2}{x} \, dx = \]
(a) \( \frac{e^2}{2} - \frac{2}{e^2} \) (b) \( \frac{e}{2} - \frac{2}{e} \) (c) \( \frac{2e}{3} \) (d) \( \frac{2}{3} \) (e) \( \frac{e^2}{2} - \frac{1}{2e^2} \) (f) \( \frac{2e^2}{3} \)

13. What is the average value of the function \( f(x) = \sqrt{\arcsin \frac{x}{1-x^2}} \) on the interval \([0, \frac{1}{2}]\)?
\[
\begin{array}{ll}
(a) \frac{\pi \sqrt{6}}{15} & \text{ (b) } \frac{\pi^{3/2} \sqrt{2}}{54} \\
(c) \frac{\pi \sqrt{3}}{18} & \text{ (d) } \frac{\pi^{3/2} \sqrt{6}}{27} \\
(e) \frac{\pi \sqrt{5}}{48}
\end{array}
\]
14. What is the volume of the solid obtained by rotating the region bounded by the graphs of \( y = \sqrt{x}, \ y = 2 - x \) and \( y = 0 \) around the \( x \)-axis?

(a) \( \frac{5\pi}{6} \)  (b) \( \frac{2\pi}{3} \)  (c) \( \frac{2\pi}{5} \)  (d) \( \frac{11\pi}{6} \)  (e) \( \frac{5\pi}{3} \)  (f) \( \frac{7\pi}{6} \)

15. The functions \( \cosh x \) and \( \sinh x \) are defined as \( \cosh x = \frac{1}{2}(e^x + e^{-x}) \) and \( \sinh x = \frac{1}{2}(e^x - e^{-x}) \). Calculate the area bounded by the curves \( y = \cosh x, \ y = \sinh x, \ x = 0 \) and \( x = b \). What is the limit of this area as \( b \to \infty \)?

(a) 1  (b) \( e \)  (c) \( e + \frac{1}{e} \)  (d) \( \frac{1}{e} \)  (e) \( \infty \)

16. Suppose \( f(x) \) is a monotonically increasing function on the interval \([0, \infty)\), and that \( f(0) = 2 \) and \( \lim_{x \to \infty} f(x) = 6 \). Suppose \( A(b) \) is the average value of \( f(x) \) on the interval \([0, b]\) for \( b > 0 \). What is \( \lim_{b \to \infty} A(b) \)?

(a) 2  (b) 4  (c) 0  (d) 6  (e) 5  (f) \( \infty \)

17. What is the volume obtained by revolving the region bounded by \( y = x^2 - 4 \) and \( y = 4 - x^2 \) around the line \( x = 2 \)?

(a) \( \frac{16\pi}{3} \)  (b) \( \frac{64\pi}{3} \)  (c) \( \frac{256\pi}{3} \)  (d) \( \frac{512\pi}{3} \)  (e) \( \frac{1024\pi}{3} \)

18. \( \int_{\pi/4}^{\pi/2} \cos \frac{\sqrt{x}}{\sqrt{x}} \, dx = \)

(a) 0  (b) 1  (c) \( \sqrt{2} \)  (d) 2  (e) \( 2\sqrt{2} \)  (f) 3

19. Compute \( \int (\sin^2 x + 8)^7 \sin x \cos x \, dx \).

(a) \( \frac{1}{16}(\sin^2 x + 8)^8 + C \)  (b) \( (\cos^2 x + 8)^7 + C \)  (c) \( \frac{1}{2}(\sin^2 x + 8)^8 + C \)

(d) \( \frac{1}{8}(\sin^2 x + 8)^8 + C \)  (e) \( \frac{1}{8}(\cos^2 x + 8)^8 + C \)  (f) \( \frac{1}{32}(\sin^2 x + 8)^7 + C \)

20. Suppose \( f \) is a continuous function and \( \int_{1}^{9} f(x) \, dx = 6 \). Then \( \int_{1}^{3} xf(x^2) \, dx = \)

(a) 6  (b) 4  (c) 3  (d) 2  (e) 1  (f) 0
21. \[ \int_{0}^{1} \frac{e^{\arctan(x)}}{1 + x^2} \, dx = \]
(a) \( e^{\pi/4} - 1 \) (b) \( e^{\pi/4} \) (c) \( e - 1 \) (d) \( \frac{\pi}{4} \) (e) \( \frac{\pi}{4} - e \) (f) \( e^{\pi/2} - e^{\pi/4} \)

22. Find the area between the graphs of \( y = \sin x \) and \( y = \cos x \) for \( 0 \leq x \leq \pi/4 \)
(a) \( \frac{\sqrt{2}}{2} \) (b) \( \sqrt{2} \) (c) \( \sqrt{2} - 1 \) (d) \( \frac{\sqrt{2}}{2} + 1 \) (e) \( \frac{\sqrt{2}}{2} - 1 \) (f) 1

23. \[ \int_{0}^{\pi/3} \sec^3 x \tan x \, dx \]
(a) \( \frac{\sqrt{2}}{6} - \frac{1}{6} \) (b) \( \frac{\sqrt{3}}{6} - 1 \) (c) \( \frac{11}{3} \) (d) \( \frac{7}{3} \) (e) \( \frac{\sqrt{2}}{3} - \frac{1}{3} \) (f) \( \frac{\sqrt{3}}{2} - \frac{1}{2} \)

24. The base of a solid is the triangle in the \( xy \)-plane with vertices (0,0), (1,0) and (0,1). Cross-sections of the solid perpendicular to the \( x \)-axis are squares. What is the volume of the solid?
(a) \( \frac{2}{3} \) (b) \( \frac{1}{4} \) (c) \( \frac{3}{4} \) (d) \( \frac{4}{3} \) (e) \( \frac{5}{4} \) (f) \( \frac{1}{3} \)

25. Find the volume of the solid obtained by rotating the area between the graphs of \( y = x^2 \) and \( x = 2y \) around the \( y \)-axis.
(a) \( \frac{2\pi}{45} \) (b) \( \frac{\pi}{2} \) (c) \( \frac{\pi}{96} \) (d) \( \frac{\pi}{24} \) (e) \( \frac{\pi}{180} \) (f) \( \frac{3\pi}{64} \)

26. \[ \int_{1}^{2} \ln\frac{x}{x^2} \, dx = \]
(a) \( 4 - \ln(\sqrt{2}) \) (b) \( \frac{1 - \ln(2)}{2} \) (c) \( \ln(2) - 1 \) (d) \( \ln(2) \) (e) \( 1 - \frac{\ln(2)}{2} \) (f) \( \frac{\ln(2)}{2} \)

27. Find the average value of the function \( f(x) = t\sqrt{16 + t^2} \) on the interval \( 0 \leq x \leq 3 \).
(a) \( \sqrt{2} \) (b) \( \frac{2\sqrt{2}}{9} \) (c) \( \frac{32}{9} \) (d) \( \frac{61}{3} \) (e) \( \frac{\sqrt{2}}{3} \) (f) \( \frac{61}{9} \)

28. Find the volume obtained by rotating the region between the \( x \)-axis, the \( y \)-axis and the line \( x + y = 1 \) around the line \( x = -2 \).
(a) \( \frac{7\pi}{3} \) (b) \( \frac{7\pi}{6} \) (c) \( \frac{19\pi}{3} \) (d) \( \frac{19\pi}{6} \) (e) \( \frac{19\pi}{12} \) (f) \( \frac{7\pi}{12} \)
29. Find the volume of the solid obtained by rotating the region between the graph of \( y = \sin(x/2) \) and the \( x \)-axis for \( 0 \leq x \leq 2\pi \) around the \( y \)-axis.

(a) \( \frac{\pi^2}{4} \)  (b) \( \frac{\pi^2}{2} \)  (c) \( \pi^2 \)  (d) \( 2\pi^2 \)  (e) \( 4\pi^2 \)  (f) \( 8\pi^2 \)

30. Find the volume of the solid obtained by rotating the region between the graph of \( y = \sin(x/2) \) and the \( x \)-axis for \( 0 \leq x \leq 2\pi \) around the \( y \)-axis.

(a) \( \frac{\pi^2}{8} \)  (b) \( \frac{\pi^2}{4} \)  (c) \( \frac{\pi^2}{2} \)  (d) \( \pi^2 \)  (e) \( 2\pi^2 \)  (f) \( 4\pi^2 \)

1. Find the area between the graphs of \( y = 1 \) and \( y = x^4 \).

(a) \( \frac{8}{7} \)  (b) \( \frac{16}{9} \)  (c) \( \frac{16}{5} \)  (d) \( \frac{12}{7} \)  (e) \( \frac{8}{5} \)  (f) \( 1 \)

2. Find the volume of the solid obtained by rotating the area between the graphs of \( y = x \sqrt{2-x} \) and \( y = 0 \) around the \( x \)-axis.

(a) \( \frac{\pi}{30} \)  (b) \( \frac{\pi}{24} \)  (c) \( \frac{4\pi}{3} \)  (d) \( \frac{\pi}{12} \)  (e) \( \frac{3\pi}{64} \)  (f) \( \frac{\pi}{180} \)

3. Find the volume of the solid obtained by rotating the area between the graph of \( y = x \cos x \) and the \( x \)-axis for \( 0 < x < \frac{\pi}{2} \) around the \( y \)-axis.

(a) \( \frac{\pi^3}{2} \)  (b) \( \frac{2\pi^3}{3} \)  (c) \( \frac{\pi^3}{2} - 8\pi \)  (d) \( 2\pi^3 - 8\pi \)  (e) \( 2\pi^3 - 4\pi \)  (f) \( \frac{\pi^3}{2} - 4\pi \)

4. Find the volume obtained by rotating the square with corners at the points \((0,0), (0,1), (1,1)\) and \((1,0)\) around the line \( x = 4 \).

(a) \( 3\pi \)  (b) \( 5\pi \)  (c) \( 7\pi \)  (d) \( 9\pi \)  (e) \( 11\pi \)  (f) \( 13\pi \)

5. Let \( V(b) \) be the volume obtained by rotating the area between the \( x \)-axis and the graph of \( y = \frac{1}{x^3} \) from \( x = 1 \) to \( x = b \) around the \( x \)-axis. What is \( \lim_{b \to \infty} V(b) \)?

(a) \( \frac{\pi}{5} \)  (b) \( \frac{\pi}{4} \)  (c) \( \frac{\pi}{3} \)  (d) \( \frac{\pi}{2} \)  (e) \( \pi \)  (f) \( \infty \)

6. Let \( V(a) \) be the volume obtained by rotating the area between the \( x \)-axis and the graph of \( y = \frac{1}{x^{3/2}} \) from \( x = a \) to \( x = 1 \) around the \( y \)-axis. What is \( \lim_{a \to 0^+} V(a) \)?

(a) \( \frac{\pi}{5} \)  (b) \( \frac{4\pi}{3} \)  (c) \( \frac{2\pi}{3} \)  (d) \( 4\pi \)  (e) \( \frac{6\pi}{5} \)  (f) \( \infty \)
7. \[ \int_0^1 \arcsin x \, dx \]
(a) \( \frac{\pi}{3} - \frac{1}{3} \)  
(b) \( \frac{\pi}{6} - \ln 2 \)  
(c) \( \frac{\pi}{2} - 1 \)  
(d) \( \frac{\pi}{4} - \frac{\ln 2}{2} \)  
(e) \( \pi - \frac{\ln 2}{6} \)  
(f) \( \frac{\pi}{4} - \frac{1}{2} \)

8. \[ \int_0^{\sqrt{\pi}} x \sin^2(x^2) \, dx \]
(a) \( \pi \)  
(b) \( \frac{\pi}{2} \)  
(c) \( \frac{\pi}{3} \)  
(d) \( \frac{2\pi}{3} \)  
(e) \( \frac{\pi}{4} \)  
(f) \( \frac{3\pi}{4} \)

9. Find the area between the graphs of \( y = x^4 + 4x^2 \) and \( y = 4x^3 \).
(a) \( \frac{1}{30} \)  
(b) \( \frac{8}{15} \)  
(c) \( \frac{16}{15} \)  
(d) \( \frac{27}{10} \)  
(e) \( \frac{81}{10} \)  
(f) \( \frac{512}{15} \)

10. The average value of the function \( f(x) \) on the interval \( [0, b] \) is \( \sqrt{b} \). What is \( f(x) \)?
(a) \( (1 + x)e^x \)  
(b) \( \frac{e^x}{1 + x} \)  
(c) \( \cos x + x \sin x \)  
(d) \( \sin x + x \cos x \)  
(e) \( \frac{3}{2\sqrt{x}} \)  
(f) \( \frac{3\sqrt{x}}{2} \)

31. Calculate the length:
(a) of the part of \( y = \frac{1}{3}(x^2 + 2)^{3/2} \) from \( x = 0 \) to \( x = 1 \). (Answer: \( 4/3 \))
(b) of the part of \( y = x^{2/3} \) from \( x = -1 \) to \( x = 8 \) (careful!) (Answer: \( \frac{80\sqrt{10} + 13\sqrt{13} - 16}{27} \))
(c) of the part of \( y = \ln(\cos x) \) for \( 0 \leq x \leq \pi/4 \). (Answer: \( \ln(\sqrt{2} + 1) \))
(d) of the part of \( y = \ln x \) for \( 1 \leq x \leq 2 \) (Answer: \( \sqrt{5} - \sqrt{2} + \frac{1}{2} \ln(\sqrt{5} - 1) - \frac{1}{2} \ln(\sqrt{2} - 1) - \frac{1}{2} \ln(\sqrt{5} + 1) + \frac{1}{2} \ln(\sqrt{2} + 1) \))

32. Calculate the surface area obtained by rotating:
(a) \( y = \sqrt{x} \) around the \( x \) axis for \( 0 \leq x \leq 4 \) (Answer: \( \frac{\pi}{6}(17\sqrt{17} - 1) \))
(b) \( y = x^3 \) around the \( x \) axis for \( 1 \leq x \leq 2 \) (Answer: \( \frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10}) \))
(c) \( y = \sqrt{9 - x^2} \) around the \( y \) axis for \( 1 \leq x \leq 3 \). (Answer: \( 12\pi\sqrt{2} \))
(d) \( y = x^2 \) around the \( x \) axis for \( 0 \leq x \leq 1 \). (Answer: \( \frac{\pi}{32}(18\sqrt{5} - \ln(\sqrt{5} + 2)) \))