Math 114-004 Spring 2011: Exam 1

Duration of exam: 70 minutes

10th February, 2011

Name: ________________________________

Recitation time (underline):
Mon. 0900–1000 / Mon. 1000–1100 / Wed. 0800–0900 / Wed. 0900–1000

Instructions:
1. There are a total of 15 multiple-choice questions and the maximum marks is 150 with all questions carrying equal credit. To get credit, you must show full work and choose the correct answer. Just choosing the correct answer with no or incorrect work will earn you NO marks. Partial credit will be awarded depending on the work shown. Please mark your answer by circling one of the choices listed below the question.

2. You are allowed to keep with you a formula-sheet which should be one standard letter-sized sheet (both sides of which can be used). You are NOT allowed to use books, class notes or any calculating aids.

3. If you have any other doubts regarding the question paper, please feel free to ask me.

4. IMPORTANT: Use of unfair means will not be tolerated at any cost–they will be reported to the Department and/ or the Office of Student Conduct for further action to be taken.

ALL THE BEST!
1. The sphere with centre $(3, 6, 3)$ and radius 5 units intersects the plane $z = 0$ in:

(a) an ellipse with major axes 6 and minor axes 3.
(b) a parabola.
(c) a hyperbola.
(d) a circle with radius 4.
(e) a pair of parallel straight lines.
(f) a pair of intersecting straight lines.
2. Which of the following expressions make mathematical sense? You are not required to evaluate them.

I. \((2i + 3j - 6k) \times (5i + 7j - 4k)) \cdot ((7i + 9j - k) \times (3i + 8j + 4k))\).

II. \(((2i + 3j - 6k) \times (5i + 7j - 4k)) \times ((7i + 9j - k) \times (3i + 8j + 4k))\).

III. \(((2i + 3j - 6k) \times (5i + 7j - 4k)) + ((7i + 9j - k) \cdot (3i + 8j + 4k))\).

(a) I, II and III.
(b) I and II only.
(c) II and III only.
(d) I and III only.
(e) None of them.
(f) III only.
3. Find the value(s) of $a$ for which the points $(1, 2, 2)$, $(2, -1, 4)$, $(3, 4, -2)$ and $(a, 6, 1)$ are coplanar i.e., all lie on one plane.

(a) 1.
(b) 7, −3.
(c) $a$ can be any real number.
(d) 0.
(e) −2.
(f) 0, 1.
4. The equation of the plane passing through (4, 2, 1) and parallel to the plane \(2x - y + 3z = 1\) is:

(a) \(3x + y - 3z = 0\).
(b) \(2x - y + 3z = 0\).
(c) \(4x - y + 2z + 9 = 0\).
(d) \(4x - 2y + 6z = 18\).
(e) \(4x + 2y + z = 0\).
(f) \(x - 2y + 3z = 7\).
5. The coordinates of the point on the plane \( x + 2y - z = 11 \) which is closest to the point \((3, 1, 7)\) is:

(a) \((9, 2, 2)\).
(b) \(\left(\frac{3}{2}, 6, \frac{1}{2}\right)\).
(c) \((1, 2, -8)\).
(d) \(\left(\frac{31}{6}, \frac{16}{3}, \frac{29}{6}\right)\).
(e) \((8, 2, 1)\).
(f) \((3, 1, 7)\).
6. If $|\mathbf{u}| = 5$ and $|\mathbf{v}| = 6$, then the maximum possible value of $|\mathbf{u} \times \mathbf{v}|$ is:

(a) 25.
(b) 36.
(c) 18.
(d) 30.
(e) 12.
(f) 27.
7. The distance of the point of intersection of the straight lines \( x = 1 + t, \\
y = 3 - t, z = 2 + t \) and \( x = 3 + 7s, y = 1 - 8s, z = 4 + 4s \) from the plane \( 2x + 2y - z + 2 = 0 \) is:

(a) 6.
(b) -6.
(c) 2.
(d) 3.
(e) 1.
(f) 5.
8. The angle between the two planes $2x + 2y + z - 11 = 0$ and $x + 3y - 8z = 4$ is:

(a) $\frac{\pi}{4}$ radians.
(b) $\frac{\pi}{6}$ radians.
(c) $\frac{\pi}{3}$ radians.
(d) $\frac{\pi}{4}$ radians.
(e) The planes are parallel.
(f) They are equations of the same plane.
9. A parametrisation of the circle $1 = 4x^2 + 4y^2$ is:

(a) $x = 4 \cos t, \ y = 4 \sin t, \ 0 \leq t \leq 2\pi$.
(b) $x = 2 \cos t, \ y = 2 \sin t, \ 0 \leq t \leq 2\pi$.
(c) $x = \cos t, \ y = \sin t, \ 0 \leq t \leq 2\pi$.
(d) $x = \frac{1}{2} \cos t, \ y = \frac{1}{2} \sin t, \ 0 \leq t \leq 2\pi$.
(e) $x = \frac{1}{2}, \ y = \frac{1}{2}$.
(f) $x = \frac{t}{2}, \ y = \frac{t}{2}, \ 0 \leq t \leq 2\pi$. 
10. The domain of the vector function

\[ f(t) = < \sqrt{4-t^2}, e^{-3t}, \ln(1+t) > \]

is:

(a) \{t \mid 1 \leq t < 2\}.
(b) \{t \mid t \leq -2\}.
(c) \{t \mid t \geq 2\}.
(d) \{t \mid 1 < t \leq 2\}.
(e) \{t \mid 2 \leq t \leq 2\}.
(f) \{t \mid 2 \leq t\}.
11. Let $\ell_1$ be the straight line with parametric equation $x = 1 + 2t, y = 2 + 3t, z = 3 + 4t$ and $\ell_2$ be the straight line with parametric equation $x = -1 + 6s, y = 3 - s, z = -5 + 2s$. Then, $\ell_1$ and $\ell_2$: 

(a) intersect at one point.
(b) are parallel.
(c) are skew.
(d) are the same straight line.
(e) are skew and intersect.
(f) are parallel and skew.
12. Let \( \mathbf{a} \) and \( \mathbf{b} \) be any two vectors in \( \mathbb{R}^3 \). Look carefully at the following identities:

I. \( \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{a} = 0 \).
II. \( \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} = 0 \).
III. \( \mathbf{a} \cdot \mathbf{b} - \mathbf{b} \cdot \mathbf{a} = 0 \).
IV. \( \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0 \).

Which of the above hold true?

(a) I, II, III and IV.
(b) I, II and IV only.
(c) I, III and IV only.
(d) II only.
(e) I and III only.
(f) I and II only.
13. The equation of the plane that passes through the line of intersection of the planes \( x - z = 1 \) and \( y + 2z = 3 \) and is perpendicular to the plane \( x + y - 2z = 1 \) is:

(a) \( 2x + y - 3z = 4 \).
(b) \( x + 6y - z = 7 \).
(c) \( x + y + z = 4 \).
(d) \( 3x - z = 9 \).
(e) \( 6x + 2y - z = 11 \).
(f) \( x - y - z = 9 \).
14. The area of the triangle $PQR$ with vertices $P(2, 1, 5)$, $Q(-1, 3, 4)$ and $R(3, 0, 6)$ is:

(a) $\sqrt{3}$.
(b) $\frac{3}{2}$.
(c) 11.
(d) $3\sqrt{3}$.
(e) $\frac{3}{\sqrt{2}}$.
(f) $\sqrt{6}$.
15. Let $S$ be the quadric determined by the equation:

$$\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} = 1.$$ 

(a) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are ellipses. The graph of $S$ is an ellipsoid.

(b) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are parabola, hyperbola and ellipse respectively. The graph of $S$ is an elliptic paraboloid.

(c) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are circles. The graph of $S$ is a sphere.

(d) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are ellipse, hyperbola and hyperbola respectively. The graph of $S$ is a cone.

(e) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are hyperbola, parabola and circle. The graph of $S$ is a hyperbolic paraboloid.

(f) Traces of $S$ on the $xy$-, $yz$- and $xz$-planes are ellipse, hyperbola and hyperbola. The graph of $S$ is a two-sheeted hyperboloid.