1. For which values of $k$ does the function $z = kx^2 - 2xy + 4y^2$ have a local minimum at $(0,0)$?

(A) $k > 0$  
(B) $k < 0$  
(C) $k > \frac{1}{4}$  
(D) $k > 4$  
(E) for no values of $k$

2. $\int_1^5 \int_0^{\ln x} f(x,y) \, dy \, dx$ is equivalent to which of the following?

(A) $\int_1^5 \int_0^{\ln x} f(x,y) \, dx \, dy$  
(B) $\int_0^{e^5} \int_{e^y}^5 f(x,y) \, dx \, dy$  
(C) $\int_0^{\ln 5} \int_{e^y}^5 f(x,y) \, dx \, dy$

(D) $\int_0^{\ln x} \int_1^5 f(x,y) \, dx \, dy$  
(E) $\int_1^{e^5} \int_0^{e^y} f(x,y) \, dx \, dy$

3. Find the directional derivative of the function $f(x,y) = \sqrt{xy}$ at $(1,4)$ in the direction of a (unit) vector making an angle of $\pi/3$ with the positive $x$-axis.

(A) $\frac{1}{2} + \frac{\sqrt{3}}{8}$  
(B) $\frac{1}{8} + \frac{\sqrt{3}}{2}$  
(C) $\frac{1}{4} + \frac{\sqrt{3}}{2}$

(D) $\frac{1}{3} + \frac{\sqrt{2}}{4}$  
(E) $\frac{1}{8} + \frac{\sqrt{3}}{4}$

4. Calculate $\int \int_R y \, dA$ over the region in the first quadrant between the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

(A) 1  
(B) 4/3  
(C) 5/3

(D) 2  
(E) 8/3

5. Let $R$ be the region in the plane above the parabola $y = x^2$ and below the line $y = x + 6$. Calculate the average value of the function $f(x,y) = x$ over $R$. (This is equal to the $x$-coordinate of the center of mass of a uniform plate that covers $R$). Recall that the average value of a function $f$ on a two-dimensional region $R$ is equal to the integral of $f$ over the region, divided by the area of the region.

(A) 0  
(B) 1/4  
(C) 1/3

(D) 1/2  
(E) 2/3

6. Suppose $f(x,y,z) = ye^{xz}$. A unit vector $\mathbf{V}$ in the direction of fastest decrease of $f$ at the point $(1,1,0)$ is

(A) $\mathbf{V} = -\mathbf{i}$  
(B) $\mathbf{V} = \mathbf{j} + \mathbf{k}$

(C) $\mathbf{V} = -\mathbf{j} - \mathbf{k}$  
(D) $\mathbf{V} = \frac{1}{\sqrt{2}}(\mathbf{j} + \mathbf{k})$

(E) $\mathbf{V} = -\frac{1}{\sqrt{2}}(\mathbf{j} + \mathbf{k})$
7. Find the (absolute) maximum and minimum values of the function \( f(x, y) = x^2 + 2y^2 - 2x \) on the closed disk \( x^2 + y^2 \leq 4 \).

(A) Maximum: 1, Minimum: 8  
(B) Maximum: 0, Minimum: 9  
(C) Maximum: -1, Minimum: 9  
(D) Maximum: -8, Minimum: 8  
(E) Maximum: -9, Minimum 9

8. \( \int_0^2 \int_0^{2x-x^2} f(x, y) \, dy \, dx \) is equivalent to which of the following?

(A) \( \int_0^2 \int_0^{2y-y^2} f(x, y) \, dx \, dy \)  
(B) \( \int_0^1 \int_0^{2y-y^2} f(x, y) \, dx \, dy \)  
(C) \( \int_0^1 \int_0^{1+\sqrt{1-y}} f(x, y) \, dx \, dy \)

(D) \( \int_0^1 \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} f(x, y) \, dx \, dy \)  
(E) \( \int_0^1 \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} f(x, y) \, dx \, dy \)

9. Calculate \( \int \int_D e^{-x^2-y^2} \, dA \) where \( D \) is the disk of radius 2 centered at the origin (i.e., the set where \( x^2 + y^2 \leq 4 \)). (Hint: It may be easiest to do this in polar coordinates)

(A) \( \pi/e^4 \)  
(B) \( 1 - \pi/e^4 \)  
(C) \( \pi - 1/e^4 \)

(D) \( \pi - \pi/e^4 \)  
(E) \( \pi/(1 + e^4) \)

10. Which of the following statements is true about the function \( f(x, y) = x^4 - 4xy + y^4 \) ?

(A) It has a local maximum, two local minima, and no other critical points.  
(B) It has a saddle point, two local maxima, and no other critical points.  
(C) It has a local minimum, a local maximum and a saddle point, and no other critical points.  
(D) It has a local minimum and two saddle points and no other critical points.  
(E) It has a saddle point, two local minima and no other critical points.

11. The shortest distance from the surface \( y^2 - xz = 5 \) to the origin is:

(a) 0  
(b) 1  
(c) \( \sqrt{5} \)  
(d) \( \sqrt{10} \)  
(e) \( \sqrt{5}/2 \)

12. What is the volume of the region contained below the paraboloid \( z = 4 - x^2 - y^2 \) and above the \( xy \)-plane?

(a) \( 2\pi \)  
(b) \( 4\pi \)  
(c) \( 8\pi \)  
(d) \( 16\pi \)  
(e) \( 32\pi \)
13. Let \( f(x, y) = \frac{x}{x + y} \). Find a unit vector \( \mathbf{u} \) such that the directional derivative of \( f \) in the direction of \( \mathbf{u} \) at the point \((3, 2)\) is zero.

(A) \( \mathbf{V} = -\mathbf{j} \)  
(B) \( \mathbf{V} = 3\mathbf{i} + 2\mathbf{j} \)  
(C) \( \mathbf{V} = \frac{1}{\sqrt{29}} (2\mathbf{i} + 5\mathbf{j}) \)  
(D) \( \mathbf{V} = \frac{1}{\sqrt{13}} (3\mathbf{i} + 2\mathbf{j}) \)  
(E) \( \mathbf{V} = -\frac{1}{\sqrt{2}} (\mathbf{i} + \mathbf{j}) \)

14. Reverse the order of integration and evaluate
\[
\int_0^\pi \int_{\sqrt{y}}^{\sqrt{x}} \frac{x^2}{x} \sin^2 x \, dx \, dy.
\]

(A) \( \frac{1}{2} \)  
(B) 1  
(C) \( \sqrt{3}/2 \)  
(D) \( 2\sqrt{3}/2 \)  
(E) \( 2\pi \)

15. The graph of the function \( z = xy^2 - x^2y + 3x \) has
   A. No critical points at all.
   B. One local maximum and one saddle point.
   C. One local maximum and one local minimum.
   D. One local minimum and one saddle point.
   E. Two saddle points.

16. \( \int_0^4 \int_{\sqrt{y}}^{2} \sqrt{1 + x^3} \, dx \, dy = \)

(a) \( \frac{52}{9} \)  
(b) \( \frac{115}{6} \)  
(c) \( \sqrt{17} + \frac{1}{3} \)  
(d) \( 10 - \sqrt{105}/2 \)  
(e) 18

17. The average value of \( x \) inside the cardioid given by \( r = 1 + \cos \theta \) is

(a) 0  
(b) \( \frac{1}{2} \)  
(c) \( \frac{5}{6} \)  
(d) 1  
(e) \( \frac{7}{6} \)

18. Which of the following shares a tangent plane with \( z = x^2 + 2y^2 \) at the point \((2,1,6)\)?

(A) \( z = xy + 3x + 2y - 4 \)  
(B) \( z = xy + 2x - 2y + 2 \)  
(C) \( z = xy + 4x + 4y - 8 \)  
(D) \( z = xy + 2x - 2y + 3 \)  
(E) \( z = xy + 3x + 2y + 4 \)

19. Use differentials to approximate the change in \( f(x, y) \) as \((x, y)\) varies from \( P \) to \( Q \), where \( f(x, y) = \ln(\sqrt{1 + xy}) \), \( P = (0, 2) \) and \( Q = (-0.09, 1.98) \).

20. The equation of the tangent plane to the graph of \( z = \frac{2}{\sqrt{xy}} + \frac{x}{y} \) at the point \((4, 1, 5)\) is
(A) $z = 2x + y + 9$  
(B) $z = \frac{7}{8}x - \frac{9}{2}y + 6$  
(C) $z = \frac{9}{5}x - \frac{7}{2}y + 4$

(D) $z = \frac{1}{4}x - \frac{3}{5}y + \frac{5}{2}$  
(E) $z = 2x - \frac{5}{4}y - \frac{7}{4}$

21. Suppose

$$w = x^2 + y^2 + z^2 \quad \text{and} \quad z^3 - xy + yz + y^3 = 1$$

define $z$ and $w$ as functions of $x$ and $y$. Then when $(x, y, z) = (2, -1, 1)$, the value of $\partial w/\partial x$ is

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 5

22. Suppose the equations

$$w = x^2 + y - z + \sin(x + y), \quad x + y = t$$

define $w$ and $t$ as functions of $x$, $y$ and $z$. Calculate $\partial w/\partial x$. 