Make sure you can answer all of the True-False review questions at the end of sections 5.2 and 5.6 of the textbook. Also, make sure you can do the “core problems” (section 5.6: 15, 28, 29, 31, 34, 36).

Then write up solutions to the following to hand in on Tuesday February 24:

1. For which real numbers $x$ do the vectors $(x, 1, 1, 1), (1, x, 1, 1), (1, 1, x, 1), (1, 1, 1, x)$ not form a basis of $\mathbb{R}^4$? For each of the values of $x$ that you find, what is the dimension of the subspace of $\mathbb{R}^4$ that they span?

2. (a) Find a $2 \times 2$ matrix that rotates the plane by $45$ degrees ($45$ degrees means $45$ degrees counterclockwise).

(b) Find a $2 \times 2$ matrix that rotates the plane by $45$ degrees followed by a reflection across the horizontal axis.

(c) Find a $2 \times 2$ matrix that reflects across the horizontal axis followed by a rotation of the plane by $45$ degrees.

(d) Find a matrix that rotates the plane through $60$ degrees, keeping the origin fixed.

(e) Find the inverse of each of these maps.

3. (a) Find a $3 \times 3$ matrix that acts on $\mathbb{R}^3$ as follows: it keeps the $x_1$ axis fixed but rotates the $x_2x_3$ plane by $60$ degrees.

(b) Find a $3 \times 3$ matrix $A$ mapping $\mathbb{R}^3 \to \mathbb{R}^3$ that rotates the $x_1x_3$ plane by $60$ degrees and leaves the $x_2$ axis fixed.

4. Find a real $2 \times 2$ matrix $A$ (other than $A = I$) such that $A^5 = I$. 
5. (a) Find a linear map of the plane, $A: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that does the following transformation of the letter $\text{F}$ (here the smaller $\text{F}$ is transformed to the larger one):

(b) Find a linear map of the plane that inverts this map, that is, it maps the larger $\text{F}$ to the smaller.

6. Let $A := \begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix}$. Compute

(a) the characteristic polynomial,
(b) the eigenvalues,
(c) one of the corresponding eigenvectors.