1. Suppose \( f \) is a function of one variable that has a continuous second derivative. Show that for any constants \( a \) and \( b \), the function

\[
u(x, y) = f(ax + by)
\]

is a solution of the PDE

\[
u_{xx}u_{yy} - u_{xy}^2 = 0.
\]

2. Give an example that shows why solutions of the wave equation \( u_{tt} = u_{xx} \) do not necessarily satisfy the maximum principle (i.e., give an example of an explicit solution of the equation for which the maximum principle does not hold).

3. Find the function \( u(x, t) \) that satisfies

\[
u_t = 2u_{xx}
\]

for \( (x, t) \in (0, 3) \times (0, \infty) \), together with the initial condition

\[
u(x, 0) = \sin \frac{\pi x}{6} + 4 \sin \frac{5\pi x}{6}
\]

for \( x \in [0, 3] \), and the boundary conditions:

\[
u(0, t) = 0 \quad \nu_x(3, t) = 0
\]

for all \( t > 0 \). (Hint: Look for “separated” solutions.)

4. Find the closed form (similar to d’Alembert’s formula) of the solution \( u(x, t) \) of the initial-boundary value problem for the semi-infinite string:

\[
u_{tt} - c^2u_{xx} = 0 \quad \text{for } x, t > 0
\]

where \( u(x, 0) = f(x) \) for \( x > 0 \), and \( u_t(x, 0) = 0 \) for \( x > 0 \), and \( u(0, t) = \alpha(t) \) for \( t \geq 0 \), where \( f \) and \( \alpha \) are \( C^2 \) functions and satisfy \( f(0) = \alpha(0) \), \( \alpha'(0) = 0 \) and \( \alpha''(0) = c^2 f''(0) \).

Verify that the solution is \( C^2 \) for all \( x, t > 0 \).