Math 425 **Dr. DeTurck**
Practice Midterm 1 **February 2010**

1. Suppose f is a function of one variable that has a continuous second derivative. Show that for any constants a and b, the function

$$u(x, y) = f(ax + by)$$

is a solution of the PDE

$$u_{xx}u_{yy} - u_{xy}^2 = 0.$$

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2. Give an example that shows why solutions of the wave equation $u_{tt} = u_{xx}$ do *not* necessarily satisfy the maximum principle (i.e., give an example of an explicit solution of the equation for which the maximum principle does not hold).

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3. Find the function $u(x, t)$ that satisfies

$$u_t = 2u_{xx}$$

for $(x, t) \in (0, 3) \times (0, \infty)$, together with the initial condition

$$u(x, 0) = \sin\frac{\pi x}{6} + 4\sin\frac{5\pi x}{6}$$

for $x \in [0, 3]$, and the boundary conditions:

$$u(0, t) = 0 \qquad u_x(3, t) = 0$$

for all $t > 0$. (Hint: Look for "separated" solutions.)

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4. Find the closed form (similar to d'Alembert's formula) of the solution $u(x, t)$ of the initial-boundary value problem for the semi-infinite string:

$$u_{tt} - c^2u_{xx} = 0 \quad \text{for } x, t > 0$$

where $u(x, 0) = f(x)$ for $x > 0$, and $u_t(x, 0) = 0$ for $x > 0$, and $u(0, t) = \alpha(t)$ for $t \geq 0$, where f and α are C^2 functions and satisfy $f(0) = \alpha(0)$, $\alpha'(0) = 0$ and $\alpha''(0) = c^2f''(0)$. Verify that the solution is C^2 for all $x, t > 0$.