Math 103-920 Quiz 1

KEY

For the following questions,

$$f(x) = \begin{cases} x & if\ x < 0 \\ x^2 & if\ 0 \le x < 3 \\ 5 - x & if\ x \ge 3. \end{cases}$$

1. Find the following limits:

$$\lim_{x\to 0^-} f(x) \qquad \lim_{x\to 0^+} f(x) \qquad \lim_{x\to 0} f(x)$$

$$\lim_{x\to 3^-} f(x) \qquad \lim_{x\to 3^+} f(x) \qquad \lim_{x\to 3} f(x)$$

For one sided limits, we have:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$$
$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 = 9$$
$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 5 - x = 2$$

Since $\lim_{x\to 0^-} f(x) = 0 = \lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x) = 0$

Since $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$ $\lim_{x\to 3} f(x)$ does not exist.

2. Determine where $f(x)$ is discontinuous.

Since x is continuous everywhere, $f(x)$ is continuous for $x < 0$, since x^2 is continuous, $f(x)$ is continuous for $0 < x < 3$, and since $5 - x$ is continuous, $f(x)$ is continuous for $x > 3$.

Since $\lim_{x\to 0} f(x) = 0 = f(0)$, $f(x)$ is continuous at 0.

Since $\lim_{x\to 3} f(x)$ does not exist, $f(x)$ is NOT continuous at 3.

Therefore, the only discontinuity is at $x = 3$.