Math 103-920 Quiz 3

KEY

Show your work to receive full credit!

1. Given $x+y=\cos(y)$, find y'. We differentiate both sides:

$$\frac{d}{dx}(x+y) = 1+y'$$
$$\frac{d}{dx}(\cos(y)) = -\sin(y)\cdot y'$$

Combining to obtain $1+y' = -\sin(y)\cdot y'$. We now isolate y':

$$\begin{aligned} 1+y' &= -\sin(y)\cdot y' \\ 1 &= -\sin(y)\cdot y' - y' \\ 1 &= -(\sin(y)+1)\cdot y' \\ y' &= \frac{-1}{\sin(y)+1} \end{aligned}$$

2. Given $\cos(x+y) = x^2y$, find y'.

We differentiate both sides:

$$\frac{d}{dx}(\cos(x+y)) = -\sin(x+y)\cdot(1+y')$$
$$\frac{d}{dx}(x^2y) = 2xy+x^2y'$$

Combining to obtain $-\sin(x+y)\cdot(1+y') = 2xy+x^2y'$. We now isolate y'.

$$\begin{aligned} -\sin(x+y)\cdot(1+y') &= 2xy+x^2y' \\ -\sin(x+y)-\sin(x+y)y' &= 2xy+x^2y' \\ -\sin(x+y)-2xy &= x^2y'+\sin(x+y)y' \\ -\sin(x+y)-2xy &= (x^2+\sin(x+y))y' \\ y' &= \frac{-\sin(x+y)-2xy}{x^2+\sin(x+y)} \end{aligned}$$