Math 104-003 Quiz 3

KEY

1. Use the Comparisson Theorem to determine whether

$$\int_1^{\infty} \frac{e^{-x}}{x^3} dx$$

in convergent or divergent. Justify your answer.

Observe that

$$\frac{e^{-x}}{x^3} \leq \frac{1}{x^3}$$

for all $x \in [1, \infty)$, also,

$$\int_1^{\infty} \frac{1}{x^3} dx$$

converges. Therefore, by the comparison theorem,

$$\int_1^{\infty} \frac{e^{-x}}{x^3} dx$$

also converges.

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   (a) $\displaystyle\int_3^\infty \frac{1}{\sqrt{x}}dx$

$$\int_3^\infty \frac{1}{\sqrt{x}}dx = \lim_{t\to\infty}\int_3^t \frac{1}{\sqrt{x}}dx = \lim_{t\to\infty}\left[2\sqrt{x}\right]_3^t$$
$$= \lim_{t\to\infty}(2\sqrt{t}-2\sqrt{3}) = \infty.$$

   Therefore, this integral diverges.

   (b) $\displaystyle\int_1^\infty xe^{-x^2}dx$

$$\int_1^\infty xe^{-x^2}dx = \lim_{t\to\infty}\int_1^t xe^{-x^2}dx$$
$$\begin{bmatrix} u=x^2 & du=2xdx \\ x=1 & u=1 \\ x=t & u=t^2 \end{bmatrix} = \lim_{t\to\infty}\int_1^{t^2}\frac{1}{2}e^{-u}du$$
$$= \lim_{t\to\infty}\left[-\frac{1}{2}e^{-u}\right]_1^{t^2}$$
$$= \lim_{t\to\infty}\left(-\frac{1}{2}e^{-t^2}+\frac{1}{2}e\right) = \frac{e}{2}$$

   So, this integral converges to $e/2$.

   (c) $\displaystyle\int_1^5 \frac{1}{(x-1)^2}dx$

$$\int_1^5 \frac{1}{(x-1)^2}dx = \lim_{t\to 1^+}\int_t^5 \frac{1}{(x-1)^2}dx$$
$$= \lim_{t\to 1^+}\left[-\frac{1}{x-1}\right]_t^5 = \lim_{t\to 1^+}\left(-\frac{1}{4}+\frac{1}{t-1}\right)$$
$$= \infty$$

   So, this integral diverges.