Math 104-003 Quiz 3

KEY

1. Use the Comparisson Theorem to determine whether

$$\int_1^\infty \frac{\sin^4 x}{x^2} dx$$

in convergent or divergent. Justify your answer.

$$0 \leq \frac{\sin^4 x}{x^2} \leq \frac{1}{x^2}$$

Also,

$$\int_1^\infty \frac{1}{x^2} dx$$

converges. Therefore,

$$\int_1^\infty \frac{\sin^4 x}{x^2} dx$$

converges.

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\displaystyle\int_3^\infty \frac{1}{\sqrt{x^3}}dx$

$$\int_3^\infty \frac{1}{\sqrt{x^3}}dx = \lim_{t\to\infty}\int_3^t \frac{1}{\sqrt{x^3}}dx = \lim_{t\to\infty}\left[\frac{-2}{\sqrt{x}}\right]_3^t$$
$$= \lim_{t\to\infty}\left(\frac{-2}{\sqrt{t}}+\frac{2}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$$

This integral converges to $\frac{2}{\sqrt{3}}$.

(b) $\displaystyle\int_e^\infty \frac{1}{x\ln(x)}dx$

$$\int_e^\infty \frac{1}{x\ln(x)}dx = \lim_{t\to\infty}\int_e^t \frac{1}{x\ln x}dx$$
$$\left[\begin{array}{cc} u=\ln x & du=\frac{dx}{x} \\ x=e & u=1 \\ x=t & u=\ln t \end{array}\right] = \lim_{t\to\infty}\int_1^{\ln t}\frac{du}{u}$$
$$= \lim_{t\to\infty}[\ln u]_1^{\ln t} = \lim_{t\to\infty}\ln(\ln t)$$
$$= \infty.$$

This integral diverges.

(c) $\displaystyle\int_0^1 \frac{1}{(x-1)^2}dx$

$$\int_0^1 \frac{1}{(x-1)^2}dx = \lim_{t\to 1^-}\int_0^t \frac{1}{(x-1)^2}dx$$
$$= \lim_{t\to 1^-}\left[\frac{-1}{x-1}\right]_0^t$$
$$= \lim_{t\to 1^-}\left(\frac{-1}{t-1}-1\right)$$
$$= \infty$$

This integral diverges.