Math 114-004

Quiz 6

Name: _____________________________                Section: ________

Instructions: Please show all of your work. No calculators, notes, or talking.

1. Find the equation of the normal line to the surface
   \[ 2(x - 2)^2 + (y - 1)^2 + (z - 3)^2 = 0 \] at the point (3, 3, 5).
   The normal line is in the direction perpendicular to the surface at
   (3, 3, 5). Since the surface is given by
   \[ f(x, y, z) = 2(x - 2)^2 + (y - 1)^2 + (z - 3)^2 = 0, \]
   a normal direction is \( \nabla f(3, 3, 5) = (4, 4, 4) \). Thus the line is
   \[ (3, 3, 5) + t(4, 4, 4). \]

2. Find the points on the cone \( z^2 = x^2 + y^2 \) that are closest to the point
   (4, 2, 0).
   We need to minimize \( (x - 4)^2 + (y - 2)^2 + z^2 \). Since we are on the
   cone, we have \( z^2 = x^2 + y^2 \) so plugging this in, we need to minimize
   the function \( f(x, y) = (x - 4)^2 + (y - 2)^2 + x^2 + y^2 \). To do this we
   need to find where \( \nabla f = 0 \). We have
   \[ \nabla f = (2(x - 4) + 2x, 2(y - 2) + 2y) = (4x - 8, 4y - 4), \]
   which is zero at (2, 1). Plugging this into the equation for the cone we get \( z^2 = 5 \) so
   that \( z = \pm \sqrt{5} \). Thus the points \( (2, 1, \pm \sqrt{5}) \) are the closest to (4, 2, 0).