1. Evaluate the integral
\[ \int_0^{\pi/4} \cos^3 x \sin^3 x \, dx. \]

The trick when one of the powers is odd is to peel off one factor and use
the identity \( \sin^2 x + \cos^2 x = 1 \). Here both powers are odd so we can peel
off either one (I’ll do cos):
\[ \int_0^{\pi/4} \cos^3 x \sin^3 x \, dx = \int_0^{\pi/4} \cos^2 x \sin^3 x \cos x \, dx = \int_0^{\pi/4} (1 - \sin^2 x) \sin^3 x \cos x \, dx. \]

Now we make the substitution \( u = \sin x \) so that \( du = \cos x \, dx \). Switching
to \( u \) limits, when \( x = 0 \) we have \( u = \sin 0 = 0 \) and when \( x = \pi/4 \) we have
\( u = \sin \pi/4 = \sqrt{2}/2 \). Then the integral becomes
\[ \int_0^{\sqrt{2}/2} (1 - u^2) u^3 \, du = \int_0^{\sqrt{2}/2} u^3 - u^5 \, du = \frac{u^4}{4} - \frac{u^6}{6} \bigg|_0^{\sqrt{2}/2} = \frac{1}{24}. \]

2. Use shell method to compute volume of the solid obtained by rotating
the following region about the \( x \)-axis. The region is given by
\[ y = \ln(x + 1), \ y = 0, \ \text{and} \ 0 < x < 1. \]

The region is the region bounded by the red curve (the graph of
\( y = \ln(x + 1) \)), the blue line (\( x = 1 \)) and the \( x \)-axis (\( y = 0 \)).
To use shell method for this problem, we need to integrate along $y$.
Solving $y = \ln(x + 1)$ for $x$ gives $x = e^y - 1$. Shown in green is some $y$
value. The radius of a shell is the distance to the axis of rotation and so is $y$. The height of a shell is the length of the green line and so is $1 - (e^y - 1)$
since the distance from the blue line to the $y$-axis is 1 and the distance
from the red line at $y$ to the $y$-axis is $e^y - 1$. The $y$-value of the
intersection of the red and blue lines is $\ln 2$. Therefore

$$V = 2\pi \int_0^{\ln 2} y(1 - (e^y - 1)) \, dy = 2\pi \int_0^{\ln 2} y(2 - e^y) \, dy.$$ 

You can now use integration by parts with $u = y$ and $dv = 2 - e^y$ to finish
the problem.