Instructions: Please show all of your work. No calculators, notes, or talking.

1. Solve the initial value problem

\[ \frac{dy}{dx} = \frac{e^x}{y^2}, \quad y(0) = 1. \]

This equation is separable: cross multiplying gives \(y^2 \, dy = e^x \, dx\).
Integrating both sides gives \(\frac{1}{3} y^3 = e^x + C\). Putting in 0 for \(x\) and 1 for \(y\) gives \(\frac{1}{3} = 1 + C\) so that \(C = -2/3\). Therefore the solution is
\[ \frac{1}{3} y^3 = e^x - \frac{2}{3} \text{ or } y = \sqrt[3]{3} e^x - \frac{2}{3}. \]

2. Determine whether the following series converges or diverges. If it converges, find what it converges to.

\[ \sum_{n=2}^{\infty} \frac{1}{3^n}. \]

Recall that \(\sum_{n=1}^{\infty} r^{n-1} = \frac{1}{1-r}\). We manipulate the series to make it of this form:

\[ \sum_{n=2}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{1}{3^n} \frac{1}{3} = 1 \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1} = 1 \left[ \frac{1}{9} \frac{1}{1 - 1/3} \right] = \frac{1}{6}. \]

Note there are other (basically equivalent) ways to do this.