Math 314/514: Fall 2015, Fall 2016, Spring 2018.

Syllabus developed by Antonella Grassi.

Text materials

Linear Algebra Done Wrong, Sergei Treil, September 2014 Version
https://www.math.brown.edu/ treil/papers/LADW/LADW-2014-09.pdf

Linear Algebra, Hoffman-Kunze, Second Edition

Some parts of: Introduction to Linear Algebra, G. Strang

Notes by Antonella Grassi

Sections from the textbooks

Treil:
Ch 2: 1-7; Ch 1: 1-7; Ch 3; Ch 4; Ch 8: 1; Ch 5; Ch 6: 1-5; Ch 8: 1-4; Ch 9. If time: Chapter 7.4

Hoffman-Kunze:
Ch 1; Ch 2; Ch 3; Ch 4: 1, 2; Ch 5: 1-4,6; Ch 6; Ch 7: 1-4; Ch 8, Ch 9.

Topics

Fields, Rings. Scalars. Matrix multiplication. Linear systems and matrices. Augmented matrix. Echelon form; reduced Echelon form.

Elementary transformations. Elementary transformations are invertible. Matrix form of elementary transformations. A is (row) equivalent to B. Theorem: Ax=0 and Bx=0 have the same solutions.
Theorem: A can be reduced to B in echelon form and reduced echelon form (over a field). Examples, over fields F and over the integers.
Left inverse of a matrix; right inverse. Invertible matrices. GL(n,F), gl(n,F), SL(n,F). Properties. Theorem: If A has a left (right) inverse exists, then A is invertible. Proof (via elementary matrices and row reduction to reduced echelon form). Solutions of homogenous and non-homogeneous systems.

Vector spaces, algebras. Examples. Vector subspaces. Examples; polynomials, sl(n,F), so(n,F), symmetric matrices. Span of sets in a vector spaces. Sum of subspaces. Direct sum. Linear dependence, linear independence. Basis. Coordinates of a vector in a basis. Uniqueness. Dimension. Examples. Linear transformations. Examples. Linear transformations between finite dimensional vector spaces and the associated matrices. Examples.

Injective, surjective, bijective maps, linear transformations. Image, range, kernel, rank of a linear transformations. Left inverse, right inverse of a transformation. Examples. Extensions of a basis. Propositions and Theorem needed to show: If $\dim V = \dim W = n$ and $T : V \to W$, linear, then dim ker T+ rk T= dim V.
Space of rows, space of columns of a matrix. Properties under row reductions.

Dual vector spaces. Dual basis. Dual (adjoint) of a linear transformation; relation between the corresponding matrices in the dual basis. Examples. Example of dual spaces and dual linear transformation. The matrix of the dual transformation is the transpose of the matrix of the original transformation in the appropriate basis. Annihilator of a set; $Ann(KerT)$, $Ann(Im(T))$, properties. Examples.
Corollary: $rkA = rkA^T$. More on dual spaces and dual (adjoint) transformations. The "4 fundamental spaces". Proofs of the Theorems.

Definition of the determinant of a matrix. Discussion of the axioms (in particular over $\mathbb{Q}$, $\mathbb{C}$, $\mathbb{R}$, $\mathbb{F}_2$). First properties and examples. More properties of the Determinant functions (via row reductions). The determinant exists (via cofactor expansion). Permutations. The determinant via permutations. The determinant is unique. More properties of the determinant. Determinant and Invertible matrices. Applications. Cramer's rule.

Eigenvalues, eigenvectors, the characteristic polynomial. Eigenvectors associated to distinct eigenvalues are linearly independent. Diagonalizable matrices. Characteristic polynomials over $\mathbb{R}$ and $\mathbb{C}$. Uniqueness of factorization over $\mathbb{R}$ and $\mathbb{C}$. Examples of matrices which cannot be diagonalized over $\mathbb{C}$ and $\mathbb{R}$.

Hermitian vector spaces. Bilinear forms, inner products, non degenerate and positive definite inner products. Normed spaces. Examples. Orthogonal, orthonormal basis. Gram-Schmidt algorithm. Orthogonal projections. Least squares. Approximations. Dual spaces of Hermitian spaces. Riesz Theorem. Examples.

Self adjoint operators. Proof of the Spectral Theorem. Invariant subspaces. Isometries. Orthogonal, special orthogonal matrices. Unitary matrices, special unitary matrices. Properties of unitary and orthogonal matrices. Properties of Isometries. Relation to geometry. Normal operators.

Singular value decomposition. Examples. The geometry of singular value decompositions.

Jordan canonical form over the complex numbers: statements; minimal polynomial. Upper triangular form of a (complex) matrix. Proof of the Cayley-Hamilton theorem (weak and strong form). Primary decomposition Theorem. Proof of the Jordan Canonical form over $\mathbb{C}$. Proof of the Jordan Canonical Form over $\mathbb{R}$. More on the cyclic basis of the Jordan canonical form. Examples.

Singular Value decomposition and Jordan canonical: comparisons. If time: Sylvester's criterion for positivity.

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