Math 314/514: Fall 2015, Fall 2016, Spring 2018.

Syllabus developed by Antonella Grassi.

Text materials

Linear Algebra Done Wrong, Sergei Treil, September 2014 Version

Linear Algebra, Hoffman-Kunze, Second Edition

Some parts of: Introduction to Linear Algebra, G. Strang

Notes by Antonella Grassi

Sections from the textbooks

Treil:
Ch 2: 1-7; Ch 1: 1-7; Ch 3; Ch 4; Ch 8: 1; Ch 5; Ch 6: 1-5; Ch 8: 1-4; Ch 9. If time: Chapter 7.4

Hoffman-Kunze:
Ch 1; Ch 2; Ch 3; Ch 4: 1, 2; Ch 5: 1-4,6; Ch 6; Ch 7: 1-4; Ch 8, Ch 9.

Topics


Elementary transformations. Elementary transformations are invertible. Matrix form of elementary transformations. A is (row) equivalent to B. Theorem: Ax=0 and Bx=0 have the same solutions.

Theorem: A can be reduced to B in echelon form and reduced echelon form (over a field). Examples, over fields F and over the integers.

Left inverse of a matrix; right inverse. Invertible matrices. GL(n,F), gl(n,F), SL(n,F). Properties. Theorem: If A has a left (right) inverse exists, then A is invertible. Proof (via elementary matrices and row reduction to reduced echelon form). Solutions of homogenous and non-homogeneous systems.

Injective, surjective, bijective maps, linear transformations. Image, range, kernel, rank of a linear transformations. Left inverse, right inverse of a transformation. Examples. Extensions of a basis. Propositions and Theorem needed to show: If \( \dim V = \dim W = n \) and \( T : V \to W \), linear, then \( \dim \ker T + \text{rk } T = \dim V \).

Space of rows, space of columns of a matrix. Properties under row reductions.

Dual vector spaces. Dual (adjoint) of a linear transformation; relation between the corresponding matrices in the dual basis. Examples. Example of dual spaces and dual linear transformation. The matrix of the dual transformation is the transpose of the matrix of the original transformation in the appropriate basis. Annihilator of a set; \( \text{Ann}(\ker T), \text{Ann}(\text{im}(T)) \), properties. Examples. Corollary: \( \text{rk } A = \text{rk } A^T \). More on dual spaces and dual (adjoint) transformations. The “4 fundamental spaces”. Proofs of the Theorems.


Eigenvalues, eigenvectors, the characteristic polynomial. Eigenvectors associated to distinct eigenvalues are linearly independent. Diagonalizable matrices. Characteristic polynomials over \( \mathbb{R} \) and \( \mathbb{C} \). Uniqueness of factorization over \( \mathbb{R} \) and \( \mathbb{C} \). Examples of matrices which cannot be diagonalized over \( \mathbb{C} \) and \( \mathbb{R} \).


Singular value decomposition. Examples. The geometry of singular value decompositions.


Singular Value decomposition and Jordan canonical: comparisons. If time: Sylvester’s criterion for positivity.

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