# FRG Workshop Algebraic Geometry, Symplectic Geometry and Theoretical Physics: a conference celebrating the contribution of women researchers May 6-8, 2005

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(1) Stable Vector Bundles on Calabi-Yau 3-folds and Heterotic M-theory Compactification, Elena Andreini, S.I.S.S.A.

**Abstract:** Fourier Mukai transform is used to construct holomorphic polystable principal bundles on elliptically fibered Calabi-Yau threefolds. The structure group G is a subgroup of  $E8 \times E8$ , the gauge group of the Heterotic MTheory. After compactifying the 10D theory on a CY one gets an effective 4D theory, with gauge group the centralizer of G in  $E8 \times E8$ . The effective theory is specified by choosing the CY and the principal gauge bundle.

## (2) Symplectic Surgery in Dimensions 4 and 6, Ahmet Beyaz, UC Irvine

**Abstract:** Several examples of symplectic surgery in dimension 4 are studied extensively with many beautiful applications. Probably, the most useful one is "Symplectic normal connected sum" operation applied on 2-tori. One example of symplectic surgery in dimension 6 may be so-called conifold transition which is done by removing a 2-sphere and putting a 3-sphere in. Under what conditions can this operation be done symplectically? Can it be generalized?

(3) Legendrian Torus Links, Jennifer Dalton, Bryn Mawr College

Abstract: Torus links are topological links that can be isotoped to lie on the surface of a standardly embedded torus. We study a Legendrian version of such torus links in  $\mathbb{R}^3$  equipped with the standard contact structure. Legendrian torus knots have been completely classified by Etnyre and Honda. This poster illustrates the rich questions involved in the classification of Legendrian torus links and describes some partial classification results.

(4) Brane Superpotentials from Landau-Ginzburg Models, Eleonora Dell'Aquila, Rutgers University

Abstract: This poster is based on two papers in collaboration with S. Ashok, D.-E. Diaconescu and B. Florea. We consider a class of Calabi-Yau compactifications that admit at "small volume" a non-geometric description as Landau-Ginzburg (LG) models. At the LG point of moduli space branes wrapping holomorphic curves in the CY manifold have a simple algebraic description, which makes the topological disc amplitudes particularly simple to evaluate. These amplitudes compute the tree-level superpotential of the low energy 4D theory. Therefore, in the cases we consider, an efficient way to compute the 4D superpotential at tree-level is to translate in LG language the geometric data that specifies the brane, and then determine the superpotential from LG computations. The poster illustrates the procedure in an example.

(5) Automorphisms of Certain Projective Bundles on Toric Varieties, Amassa Fauntleroy, North Carolina State

Abstract: The purpose of this investigation is to exhibit the automorphism group of a projective bundle P(E) over a simplicial toric variety X when the bundle E is a direct sum of equvariant line bundles. This case is important in the study of moduli of complete intersections on toric varieties including projective spaces. The main result is that the automorphism group of P(E) is, up to a finite group, the semi-direct product of the automorphism group of the base and a certain subgroup of fiber preserving automorphisms. This structure is similar to the structure of the automorphism groups of rational surfaces (Hirzebruch surfaces). Applications to moduli space constructions are indicated in special cases, including Del Pezzo surfaces and certain Calabi Yau *m*-folds.

(6) Moduli Spaces – Instantons – Singularities, Elizabeth Gasparim, New Mexico State

Abstract: My poster presents:

- new theorems on moduli of vector bundles on surfaces
- new invariants for singularities
- new theorems on instanton decay and instanton moduli
- math/phys translations linking the 3 results above.

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(7) **Orbifold Cohomology for Torus Quotients**, Rebecca Goldin, George Mason University and Tara Holm, UC Berkeley

Abstract: We are interested in a class of orbifolds, obtained by the quotient of a smooth manifold with by a locally free abelian group action. The group need not be finite – we are interested in the case of a compact torus action. We describe a new way of obtaining the orbifold cohomology of these quotients, using the presentation as a quotient space. Along the way, we introduce preorbifold cohomology which generalizes to the case when the group does not act locally freely; in particular, it generalizes to the case of a symplectic manifold with a Hamiltonian torus action. This generalization produces a new method for (easily!) computing the orbifold cohomology of symplectic reductions, an interesting class of orbifolds that are not global quotients of manifolds by finite group actions. This is joint work with Allen Knutson of University of California, Berkeley.

(8) **Distinguishing Chambers of the Moment Polytope**, Rebecca Goldin, George Mason University and Tara Holm, UC Berkeley

Abstract: Given a Hamiltonian T space M, where T is a torus acting on M, the moment polytope is the image of the moment map  $\phi : M \to Lie(T)^*$ . It is a convex polytope when M is compact and connected. Singularities of this map can be recorded as codimension-1 walls within the polytope. This "cuts" the polytope up into regions of nonsingular values of the moment map. These regions of singular values are called "chambers" of the moment polytope. We describe an algebraic invariant that distinguishes these chambers. The methods involve an application of the Jeffrey-Kirwan residue formula, but the results are formulated using only geometry and equivariant cohomology. In theory, this result could be used to calculate the number of chambers in the moment polytope. Our invariant is more subtle than the diffeomorphism class of corresponding symplectic reductions, and easier to compute. This is joint work with Lisa Jeffrey at University of Toronto.

(9) Elliptic Genus on Non-Spin Manifolds with Smooth Circle Actions, Haydee Herrera, Rutgers University

**Abstract:** We prove the vanishing of various characteristic numbers such as the Agenus and the rigidity of the elliptic genus on certain manifolds admitting  $\mathbb{S}^1$  actions that are non-spin.

# (10) Cohomogeneity One Examples of Special Lagrangians in the Deformed Conifold, Marianty Ionel, McMaster University

Abstract: The cotangent bundle of the *n*-sphere has a Ricci-flat Kahler metric discovered by Stenzel. The case n = 2 is the Eguchi-Hanson metric and the metric in the case n = 3 was found earlier by Candelas and de la Ossa. In my poster I will present examples of cohomogeneity one special Lagrangians in the cotangent bundle of the 3-sphere, known in the physics literature as the deformed conifold. Some of these examples can be generalized in any dimension. I will also address the asymptotic behaviour of these submanifolds and their topology. This is joint work with M. Min-Oo.

#### (11) Links of Legendrian Unknots, Jill Jordan, Bryn Mawr College

**Abstract:** We introduce a new polynomial invariant for Legendrian links based on the technique of generating families. We calculate this invariant for two families of two-component Legendrian links. We then compare the generating family polynomials to the Chekanov polynomials, defined through holomorphic curves. For both types of links, the generating family polynomials and the Chekanov polynomials are the same.

# (12) Elliptic Surfaces in N=2 Seiberg-Witten Theory, Amy Ksir, U.S. Naval Academy

Abstract: The Seiberg-Witten solution to N = 2 supersymmetric gauge theory is one place where ideas and methods from algebraic geometry, symplectic geometry, and theoretical physics interact productively. Since the original 1994 solution of the pure SU(2) theory, many approaches have been used to find solutions for other gauge groups and matter content. These approaches have included integrable systems, geometric engineering, matrix models, and instanton calculations. Here we describe how we used one approach, sometimes called "geometric engineering" in type IIA theory, to find the solutions to three problems. We give the equation for an elliptic surface in which the three families of Seiberg-Witten curves live, and the equation for the curves in the surface for the most interesting of these three theories.

This poster is based on joint work with Stephen Naculich of Bowdoin College.

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# (13) **Toric Integrable Geodesic Flows**, Christopher Lee, University of Illinois Urbana-Champain

Abstract: The geodesic flow of a compact, connected Riemannian manifold  $(Q^n, g)$  is *toric integrable* if there exists an effective action of the *n*-torus on the punctured cotangent bundle  $T^*Q \setminus Q$  which commutes with dilations, preserves the standard symplectic form and preserves the length function  $(q, p) \mapsto ||p||_q^*$ . Toric integrable geodesic flows are known to exist for certain choices of metrics on tori and low-dimensional spheres. One might ask: is there a topological obstruction to the existence of a toric integrable geodesic flow? The answer to this question is most easily pursued using tools from contact geometry. In particular, the topology of contact toric manifolds plays a role.

(14) Paths, Tableaux, and q-Characters of Quantum Affine Algebras, Wakako Nakai, Nagoya University

Abstract: In order to investigate finite dimensional representations of quantum affine algebras, the q-characeter was introduced. In the context of the transfer matrices, it is conjectured in the cases of  $U_q\left(A_n^{(1)}\right)$  and  $U_q\left(B_n^{(1)}\right)$  that the q-characters for certain finite dimensional irreducible representations are given as the Jacobi-Trudi determinants, and these determinants can be described by using tableaux, which are the known results by Bazhanov-Reshetikhin and Kuniba-Ohta-Suzuki. Our main result is to give a tableaux description of the determinant for  $U_q\left(C_n^{(1)}\right)$  and  $U_q\left(D_n^{(1)}\right)$  in some cases. To do this, we use Gessel-Viennot's paths.

This is joint work with T. Nakanishi.

- (15) **On Parallelizable 4-Manifolds**, Nadya Shirokova, Stanford University
- (16) Linearity, Symmetry and Prediction in the Hydrogen Atom, Stephanie Singer, Philadelphia

Abstract: Geometry has many deep applications in physics. This textbook (to be published this summer by Springer Verlag in the UTM series) will allow you to teach a course on one basic, representative, sexy application: the application of representation theory to the structure of the hydrogen atom and the periodic table. This is one of the great success stories of mathematics in the 20th century, and it makes a perfect capstone course: for students who are finishing their math studies, it shows the power of the algebra and analysis they have already learned; for students continuing on in mathematics it motivates a large swath of the graduate curriculum.

## (17) Four Dimensions from Two in Symplectic Topology,

Margaret Symington, Georgia Tech

**Abstract:** This poster describes a generalization of toric geometry that is fruitful for the study of symplectic manifolds of dimension four. The idea is to view the moment map for the torus action on a closed manifold as a Lagrangian fibration. Replacing the moment map image (a polygon that classifies the toric manifold) by the base of the fibration with an induced integral affine geometry allows one to generalize to non-compact manifolds, and to broaden the class of permissible singular fibers, while maintaining the informational strength of the geometric base.

#### (18) Quantum-Type Products in Symplectic Geometry,

Mihaela Vajiac, Chapman

Abstract: In this paper gauge theory techniques and the theory of flat connections are used to show that the small quantum product is a deformation of the cup product on a symplectic manifold M in a gauge theoretical sense, and to construct a moduli space of products on a complex vector space which are associative, commutative, Frobenius, and unital.

# (19) N=1 Super Yang-Mills Domain Walls via the Extended Veneziano-Yankielowicz Theory, Frederica Vian, Nordita

Abstract: We investigate the vacuum structure of pure SU(N) N = 1 super Yang-Mills. The theory is expected to possess N vacua with associated domain walls. We show that the newly extended version of the low energy effective Lagrangian for super Yang-Mills supports the BPS domain wall solutions associated with any two vacua aligned with the origin of the moduli space. For the two color theory the domain wall analysis is complete. We also find new non BPS domain wall solutions connecting any two vacua of the underlying SU(N) super Yang-Mills theory not necessarily aligned. When two vacua are aligned with the origin of the moduli space these solutions are the BPS ones. We also discuss the generic BPS domain wall solutions connecting any two vacua within the extended Veneziano-Yankielowicz theory.

#### (20) Dirac Structures and Contact Geometry, Aissa Wade, Penn State

**Abstract:** We present characterizations of contact 1-forms in terms of Dirac structures. We also relate almost contact structures to the theory of Dirac structures.

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