Math 116 Problem Set #4 Due Wed., Sept. 26, 2018, in class.

Reminder: Exam 1 will take place in class on Friday, Sept. 28, and will cover material up through this problem set. One two-sided handwritten 5"x 7" index card is permitted during the exam, but no other materials.

In Apostol, Volume I, read Chapter 3, Sections 9-19, pages 142-154 (Sections 12-14 and 19 are optional). Also read Chapter 4, Sections 3-10, pages 159-176.

1. From Apostol, Volume I, Chapter 3, Section 3.8, page 142, do problems 12, 21.

2. From Apostol, Volume I, Chapter 3, Section 3.11, page 145, do problems 1, 5.

3. From Apostol, Volume I, Chapter 4, Section 4.6, pages 167-169, do problems 36, 38(a).

4. For each of the following functions f, determine *whether* there is an $x \in [0,1]$ such that $f(x) = 0$, and also determine *whether* f attains a maximum on $[0,1]$ at some point x in the closed interval (but in each case you are not being asked to *find* such an x). Explain your assertions and relate them to whether the Intermediate Value Theorem and the Extreme Value Theorem apply.

a) $f(x) = e^x - 2$

b) $f(x) = x + 1 - 3[x]$

c) $f(x) = 1/(2x^2 - 1)$ if x is rational, and $f(x)$ is undefined otherwise.

5. Which of the following functions on $[0,3]$ are of the form $F(x) = \displaystyle\int_a^x f$ for some f and some a? For each that is not, explain why not. For each that is, give such an f and a.

a) F is defined by $F(x) = 0$ for $0 \le x < 1$, $F(x) = x - 1$ for $1 \le x < 2$, and $F(x) = 3 - x$ for $2 \le x \le 3$.

b) F is defined by $F(x) = 1$ for $0 \le x < 1$, $F(x) = x$ for $1 \le x < 2$, and $F(x) = x - 1$ for $2 \le x \le 3$.