Math 116 Problem Set #6 Due Wed., Oct. 10, 2018, in class.

In Apostol, Volume I, read Chapter 12, Sections 10-13, pages 458-466.

1. From Apostol, Volume I, Chapter 12, Section 12.8, pages 456-457, do problems 5, 13(a,c).

2. From Apostol, Volume I, Chapter 12, Section 12.11, pages 460-462, do problems 1, 5, 18, 20. (In #20, these properties are summarized by saying that $\mathbb{R}^n$ together with this distance function is a *metric space*.)

3. From Apostol, Volume I, Chapter 12, Section 12.15, pages 467-468, do problems 1(a-c), 6, 7.

4. a) Show that if we define a new product on $\mathbb{R}^2$ by

$$(a_1, a_2) \cdot (b_1, b_2) = 2a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2,$$

then the laws of the usual dot product (see Apostol, Vol. 1, Theorem 12.2) are still satisfied.

b) In terms of coordinates, write an explicit formula for a new norm on $\mathbb{R}^2$ that is related to this new dot product by the equation $\|v\|^2 = v \cdot v$.

c) Explain why the properties of norm given in Theorems 12.4 and 12.5 are automatically satisfied for this new norm (i.e. without the need to do any new computations).

5. In $\mathbb{R}^n$ (with $n \geq 2$), show that there is *no* way to define a new dot product on $\mathbb{R}^n$ that satisfies the laws of the usual dot product and that gives the norm defined in problem 18 in Apostol, Vol. 1, Section 12.11. [Hint: Under this norm, what are the norms of e_1, e_2, and $e_1 \pm e_2$, where $e_1, \ldots, e_n$ are the unit coordinate vectors? What does this say about $e_1 \cdot e_2$ under such a new dot product?]