Math 116 Problem Set #7 Due Wed., Oct. 17, 2018, in class.

In Apostol, Volume I, read Chapter 12, Sections 14 and 16, pages 466-470; and Chapter 13, Sections 2-16, pages 472-496.

1. From Apostol, Volume I, Chapter 12, Section 12.15, pages 467-468, do problem 17; and from Section 12.17, page 470, do problem 1(a,b).

2. From Apostol, Volume I, Chapter 13, Section 13.5, page 477, do problems 1, 4(a-d); and from Section 13.8, pages 482-483, do problem 1(a-c).

3. From Apostol, Volume I, Chapter 13, Section 13.11, pages 487-488, do problems 1(c,g), 2(a), 8(a); and from Section 13.14, pages 492-493, do problems 1(b), 3.

4. Let $v_1, v_2, v_3 \in \mathbb{R}^3$. Suppose that v_1 and v_2 are non-zero orthogonal vectors, and let Π be the span of $\{v_1, v_2\}$. For $i = 1, 2$ let $a_i = v_3 \cdot v_i / \|v_i\|^2$, and let $w = a_1 v_1 + a_2 v_2$.

a) Show that Π is a plane through the origin.

b) Show that w is the orthogonal projection of v_3 onto Π; i.e. that $v_3 - w$ is orthogonal to every vector in the plane.

c) Show that w is the closest point to v_3 on Π.

d) Interpret parts (b) and (c) in the special case that v_3 lies in Π, and explain why those parts were already known by a previous result in that case.

5. a) Let L be a line in $\mathbb{R}^2$. Prove that the set of vectors in L spans $\mathbb{R}^2$ if and only if L does not contain the origin. (Note: Here we regard points in $\mathbb{R}^2$, including points on L, as vectors in the usual way, corresponding to arrows from the origin to those points.)

b) State and prove an analog for planes in $\mathbb{R}^3$.

c) What about lines in $\mathbb{R}^3$?