Math 116 Problem Set #8 Due Wed., Oct. 24, 2018, in class.

In Apostol, Volume I, read Chapter 13, Sections 18-23, pages 497-507; and Chapter 14, Sections 1-8, pages 512-528.

1. From Apostol, Volume I, Chapter 13, Section 13.21, page 503, do problems 4, 7; and from Section 13.24, pages 508-509, do problems 2, 19, 26.

2. From Apostol, Volume I, Chapter 14, Section 14.4, pages 516-517, do problems 2, 9, 14, 15; from Section 14.7, pages 524-525, do problems 4, 10, 17; and from Section 14.9, pages 528-529, do problems 4, 7.

3. a) Let L be a line in the plane and let C be a conic section in the plane. At how many points can L and C meet? Give examples illustrating each possible value.

b) In part (a), if L is tangent to C at a point P, then at how many points (including P) can L and C meet?

c) Make a conjecture concerning the number of points at which two distinct conic sections C, C' in the plane can meet. Give examples to illustrate each of the possible values.

4. Suppose that $F : \mathbb{R} \to \mathbb{R}^2$ is a differentiable vector-valued function, that $c \in \mathbb{R}$, and that $\int_0^x F(t)\,dt = (x^2 + x, e^x + c)$ for all $x \in \mathbb{R}$. Find F and find c.

5. Let $F : \mathbb{R} \to \mathbb{R}^n$ be a differentiable vector-valued function that parametrizes the motion of a particle in $\mathbb{R}^n$ whose speed is always at most c (where c is some positive real number).

a) Prove that if $a < b$ then $\|F(b) - F(a)\| \leq c(b - a)$. Also explain why this is reasonable from a geometric point of view. [Hint: for the proof, use the Fundamental Theorem of Calculus and another result.]

b) Give an example of a function F and values $a < b$ for which there is equality in part (a), and give another example in which there is a strict inequality.