Math 116 Problem Set #11 Due Wed., Nov. 14, 2018, in class.

In Apostol, Volume II, read Chapter 8, Sections 18-23, pages 269-281; and read Chapter 9, Sections 6-12, pages 294-313.

1. From Apostol, Volume II, Chapter 8, Section 8.22, pages 275-277, do problems 1, 2; and from Section 8.24, pages 281-282, do problems 1, 3.

2. From Apostol, Volume II, Chapter 9, Section 9.8, pages 302-303, do problems 1, 9; and from Section 9.13, pages 313-314, do problems 1, 2, 4, 21.

3. a) Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable functions, and let $F : \mathbb{R} \to \mathbb{R}^2$ be defined by $F(x) = (f(x), g(x))$. Define $h : \mathbb{R}^2 \to \mathbb{R}$ by $h(x, y) = xy$. Use the chain rule to compute the derivative of the composition $h \circ F : \mathbb{R} \to \mathbb{R}$. Also write out this composition as a function of x, and give another reason why the derivative of $h \circ F$ has the form you computed.

b) Let $S_1 = \{x \in \mathbb{R} \,|\, x > 0\} \subset \mathbb{R}$, and let $S_2 = \{(x, y) \in \mathbb{R}^2 \,|\, x > 0, y > 0\} \subset \mathbb{R}^2$. Define $F : S_1 \to S_2$ by $F(x) = (x, x)$. Define $g : S_2 \to \mathbb{R}$ by $g(u, v) = u^v$. Use the chain rule to compute the derivative of the composition $g \circ F : S_1 \to \mathbb{R}$. Also write out this composition as a function $h(x)$, and give another way to compute its derivative (by writing $z = h(x)$, taking logarithms of both sides, and then using implicit differentiation).

4. Let m, n be positive integers. For every i, j with $1 \leq i \leq m$ and $1 \leq j \leq n$, let $a_{i,j}$ be a real number. Define the function $F : \mathbb{R}^n \to \mathbb{R}^m$ by

$$F(x_1, \ldots, x_n) = (\sum_{j=1}^{n} a_{1,j} x_j, \ldots, \sum_{j=1}^{n} a_{m,j} x_j).$$

Show that F is differentiable, and find its total derivative (both as a linear map and in terms of its Jacobian matrix). Also find the error term.

5. For each of the following functions, determine whether it has a maximum at $(0, 0)$, a minimum at $(0, 0)$, or neither.

a) $f(x, y) = x^2 + xy + y^2$

b) $f(x, y) = x^2 + 3xy + y^2$

c) $f(x, y) = x^3 + y^3$

d) $f(x, y) = x^4 + y^4$

e) $f(x, y) = \sin(x + y) - x - y$