Math 116 Problem Set #12 Due Wed., Nov. 21, 2018, in class.

In Apostol, Volume II, read Chapter 9, Sections 14-17, pages 314-322; and Chapter 10, Sections 1-17, pages 323-345.

1. From Apostol, Volume II, Chapter 9, Section 9.15, do problems 2, 8.

2. From Apostol, Volume II, Chapter 10, Section 10.5, page 328, do problems 1, 3, 9; and from Section 10.9, pages 331-332, do problems 7, 9.

3. From Apostol, Volume II, Chapter 10, Section 10.13, pages 336-337, do problems 2, 3; and from Section 10.18, pages 345-346, do problems 2, 6, 17, 18. (Problem 17 begins on page 345.)

4. Consider the function $f(x,y) = x^2 + y^2$ on the set $S = \{(x,y) \in \mathbb{R}^2 \mid x^2 + xy + y^2 \leq 1\}$. Determine whether f achieves a maximum value on S, and whether it achieves a minimum value on S. If so, find those values and find where they are achieved. (Hint: Consider both the interior and the boundary.)

5. Consider the function $\phi(x,y) = \sin(xy) + e^{x+y}$ on $\mathbb{R}^2$.

(a) Explicitly find $F = \nabla\phi$.

(b) Let C be the curve in $\mathbb{R}^2$ with parametrization $\alpha : [0,1] \to \mathbb{R}^2$ given by

$$x = \sin(\frac{\pi}{2}t), \ y = \frac{e^t - 1}{e - 1}.$$

Evaluate $\int_C F \cdot d\alpha$. (Hint: Do not try to do it directly.)

(c) Do the same for the parametrized curve C' given by $x = t$, $y = t$. This time do it two ways: the way you did part (b), and also by evaluating it directly (using the definition of line integral). Compare your answers with each other and with your answer to part (b); what equalities hold and why?