Math 116 Problem Set #13 Due Wed., Nov. 28, 2018, in class.

In Apostol, Volume II, read Chapter 11, Sections 1-17, pages 353-377.

1. From Apostol, Volume II, Chapter 11, Section 11.9, pages 362-363, do problems 1, 4, 7, 14.

2. From Apostol, Volume II, Chapter 11, Section 11.15, pages 371-373, do problems 1, 2, 4, 6, 8(a), 10, 11, 23.

3. From Apostol, Volume II, Chapter 11, Section 11.18, pages 377-378, do problems 1, 9, 10, 19.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. For $s, t > 0$ let

$$g(s,t) = \iint_{R_{s,t}} f(x,y)\,dA,$$

where $R_{s,t}$ is the rectangle given by $0 \le x \le s$, $0 \le y \le t$. Evaluate $\partial g/\partial t$ and $\partial^2 g/\partial s \partial t$. Also interpret your answer geometrically.

5. Let m, n be positive integers. Let $P(x,y) = y^m$ and $Q(x,y) = x^n$. Let R be the square given by $0 \le x, y \le 1$, and let C be the boundary of R oriented counterclockwise.

(a) Determine if the vector field $F = P\mathbf{i} + Q\mathbf{j}$ is a gradient. (Your answer should depend on m, n.)

(b) Evaluate $\oint_C P\,dx + Q\,dy$. When is it equal to zero? Is this expected?

(c) Evaluate $\iint_R \big(\partial Q/\partial x - \partial P/\partial y\big)\,dA$. Compare your answer to the one you got in part (b).