Math 116 Problem Set #15 Due Mon., Dec. 10, 2018, in class.

Note: Those who would like to have extra time can submit this problem set to the TA by Wed., Dec. 12.

In Apostol, Volume II, read Chapter 12, Sections 1-20, pages 417-462. (Sections 16, 18, 20 are optional.)

1. From Apostol, Volume II, Chapter 12, Section 12.4, page 424, do problems 1, 8; and from Section 12.6, pages 429-430, do problems 2, 3.

2. From Apostol, Volume II, Chapter 12, Section 12.10, pages 436-438, do problems 1, 7; and from Section 12.13, pages 442-443, do problems 1, 5.

3. From Apostol, Volume II, Chapter 12, Section 12.15, pages 447-448, do problems 1(a,c), 5; and from Section 12.21, pages 462-465, do problems 1, 2.

4. Let $F = z\,\mathbf{i} + x\,\mathbf{j} + (y+z)\,\mathbf{k}$.

(a) Find $\operatorname{curl} F$ and $\operatorname{div} F$.

(b) Determine whether F is conservative; i.e., whether $\displaystyle\int_C F \cdot d\alpha$ is path independent, where α is a parametrization of C.

(c) Determine whether $F = \operatorname{curl} G$ for some vector field G.

(Hint: Use part (a) in doing parts (b) and (c).)

5. For a surface S, let $\mathbf{n}$ be the outward unit normal.

(a) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 2$ above the plane $z = 1$. Let $F = xz\,\mathbf{i} + yz\,\mathbf{j} + xy\,\mathbf{k}$. Using Stokes' Theorem, compute $\displaystyle\iint_S \operatorname{curl} F \cdot \mathbf{n}\, dS$.

(b) Let V be the solid cylinder given by $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$, and let $S = \partial V$. Let $F = 2xz\,\mathbf{i} + \sin(x^2 z)\,\mathbf{j} + (y^3 + z - z^2)\,\mathbf{k}$. Using the divergence theorem, evaluate $\displaystyle\iint_S F \cdot \mathbf{n}\, dS$.