Math 116 Sample Exam #1 September, 2018

Instructions: This is a sample exam in preparation for Exam #1, which will be held in class on Friday, Sept. 28, 2018. This sample exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points. This sample exam does not have to be handed in. But those who do hand in a completed sample exam at class on Wednesday, Sept. 26, will get extra credit.

1. Suppose that $T \subseteq S \subseteq \mathbb{R}$, and that T is not empty.
   a) Show that if S has a supremum, then so does T.
   b) Determine whether the converse also holds.

2. Let $f(x) = x^2[x]$, where $[x]$ is the greatest integer in x. Determine whether $\int_0^2 f(x)dx$ exists. If it does, evaluate it.

3. a) In terms of ε, δ, write out explicitly the meaning of the statement that the function $f(x) = 3x$ is continuous at $x = 0$.
   b) Find an explicit value of δ that works for $\varepsilon = 1/2$. Explain why this value works.

4. Let $f(x) = x^4 + x$.
   a) Must the function f achieve a maximum value on every closed interval $[a, b]$? If so, explain why. If not, give an example of a closed interval where this does not happen.
   b) Must the function f achieve a maximum value on every open interval (a, b)? If so, explain why. If not, give an example of an open interval where this does not happen.

5. Let f be a function with the property that $-x^2 \leq f(x) \leq x^2$ for all x. Determine whether f is necessarily continuous at $x = 0$. If so, explain why. If not, give a counterexample.