Math 116 Sample Exam #2 October, 2018

Instructions: This is a sample exam in preparation for Exam #2, which will be held in class on Wednesday, Nov. 7, 2018. This sample exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points. This sample exam does not have to be handed in. But those who do hand in a completed sample exam at class on Monday, Nov. 5, will get extra credit.

1. For $x \in \mathbb{R}$, let $f(x) = [x]$, the greatest integer in x. Let $g(x) = \displaystyle\int_0^x f(t)\,dt$ and let $h(x) = \displaystyle\int_0^x g(t)\,dt$.

a) Graph the functions f, g, h.

b) Determine where these functions are continuous, and where they are differentiable. Explain your assertions.

2. a) Find a vector $z \in \mathbb{R}^3$ of length 1 that is orthogonal to the vectors $v = (1, 1, 0)$ and $w = (0, 1, 1)$.

b) With v, w, z as in part (a), find the volume of the parallelepiped generated by these three vectors.

3. a) Find the focus and directrix of the parabola $y = 2x^2$.

b) Find all points of this parabola where the curvature is maximal. Is there any point at which the curvature is minimal?

4. Let $F : \mathbb{R} \to \mathbb{R}^2$ be a differentiable function. For each of the following assertions, either give a proof or a counterexample.

a) If F parametrizes a circle centered at the origin, then the acceleration vector at each point is a multiple of the position vector.

b) If the velocity is constant, then the curvature is 0.

5. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = (xy)^{1/3}$. *At the origin*, determine whether f is continuous, whether its partial derivatives exist there, whether the derivative of f there with respect to the vector $(1, 1)$ exists, and whether f is differentiable there.